

Article

Shock Waves in Ion-Beam-Depleted Spin-Polarized Quantum Plasma with Ionic Pressure Anisotropy

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Abstract: In this study, the effects of pressure anisotropy and viscosity on the propagation of shock waves in spin-polarized degenerate quantum magnetoplasma are studied under the influence of the streaming energy of ion beams. The effects of different suitable plasma parameters on the shock wave's potential profile are studied using the steady state solution of the Zakharov–Kuznetsov–Burgers (Z–K–B) equation, as well as the numerical simulation of the governing non-linear Z–K–B equation. First-order analysis of the non-linear wave propagation depicted a new beam-induced stable mode whose Mach number may be subsonic or supersonic depending on the anisotropic pressure combination in the presence of different spin density polarization ratios. This is the first observation of this new beam-induced stable mode in ion beam plasma, apart from the other existing modes of ion beam plasma systems, namely, the fast beam mode, the slow beam mode, the inherent ion acoustic mode, and the coupled mode, which also has unique propagation characteristics compared to the other modes. The spin density polarization ratio of spin-up and spin-down electrons have an unprecedented effect on the polarity and the direction of propagation of different shock wave modes in such plasma systems. Apart from the spin effect, anisotropic pressure combinations, as well as the viscosity of ions and ion beams, also play an outstanding role in controlling the nature of propagation of shock waves, especially in the newly detected beam-induced stable mode, and depending on the viscosity parameters of ions and ion beams, both oscillatory and monotonic shock waves can propagate in such plasma.

Keywords: ZK–Burgers equation; quantum plasma; viscosity; spin density polarization; pressure anisotropy



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1. Introduction

Quantum dense magnetoplasma has attracted great attention recently because of its enforcement in different plasma regimes, from laboratory to astrophysical plasma environments [1–4]. Because of its great appositeness in different practical areas of importance, such as in next-generation intense laser plasma interaction experiments [5,6], in the electrochemical systems of nanoscale regimes (such as quantum free-electron lasers, quantum

diodes, nano-plasmonics, thin metal films, and quantum dots) [7,8], and in the core of astrophysical compact objects, like neutron stars, white dwarfs, giant planets' magnetospheres, black holes, the earth's atmosphere, etc. [9,10], degenerate plasma has become an interesting research area for exploration. Inspired by the experimental works of Refs. [11,12], quite a few new challenging topics have been investigated theoretically [13,14] throughout the world in the context of dense plasma regimes. On the other hand, the spin effect of quantum electrons is an inherent property of electrons that affects plasma dynamics organically because of the intrinsic magnetic moment associated with their spinning, giving rise to several new effects. For spin- $\frac{1}{2}$ degenerate electrons, Fermi pressures are different for spin-up electrons and spin-down electrons, and thus, it has been found that the spin of electrons can modify the shape and structure of solitary waves [15]. Normally, the frequency of spin plasma waves remains in the range of cyclotron frequency. For the range of magnetic fields considered in this manuscript, the acoustic wave frequency will be quite high, constituting an exciting area of study. On the other hand, energetic ion beams are mandatory for the laboratory production of extreme states of matter which also form the interior of neutron stars, white dwarfs, etc. [1,2,16]. These ion beams are present everywhere from laboratory environments to space astrophysical plasma environments, such as in the magnetosphere of different planets [17], in supernova plasma flows, in pulsars, in blazars, etc. [18]. Therefore, the role of ion beams in plasma wave propagation is always fascinating. Moreover, thanks to the remarkable development of next-generation super-intense lasers, it has been confirmed that in the laboratory, highly dense plasma similar to that of astrophysical origin can be created [19,20]. These super-strong lasers can generate a very high magnetic field which also exists in dense astrophysical objects. Astrophysical environments and certain laboratory plasma environments have one thing in common, i.e., an intense magnetic field. For example, the magnetic field on a neutron star's surface can be of the order of $10^{11} - 10^{12}$ G, whereas its interior magnetic field can reach up to 10^{15} G; see Refs. [21,22]. On the other hand, in white dwarfs, etc., a magnetic field of the order of $10^{10} - 10^{12}$ G can exist, whereas the degenerate electron density can be of the order of $10^{26} - 10^{30} \text{ cm}^{-3}$ [23–25]. In the presence of such high magnetic fields, the anisotropic behaviour of plasma ion pressure is revealed, and plasma behaves differently in parallel and perpendicular directions relative to the external magnetic field [26]. Therefore, the effects of ionic pressure and anisotropy pressure, i.e., parallel ion pressure ($P_{\parallel}$) and perpendicular ion pressure ($P_{\perp}$), become very important. Numerous investigations have reported on the effect of pressure anisotropy on the propagation of solitary and shock waves in different plasma regimes. For example, Singh and Saini investigated the properties of electron acoustic shock waves in superthermal electron–positron plasma, including the effect of anisotropic pressure, and they found that shock potential increases with an increasing external magnetic field. They also found that shock strength increases with an increase in the superthermal parameters of both the electrons and the positrons [27]. Hau and Hung [28] studied the formation of anomalous slow shock in anisotropic plasmas and found that Magneto Hydrodynamic (MHD) equations were greatly modified by the presence of anisotropic pressure. Blokhin and Trakhinin [29], using the famous CGL (Chew, Goldberger, and Low) MHD equation, carried out a stability analysis of fast parallel and transversal MHD shock waves in anisotropic pressure plasma. Mahmood et al. [30] studied the properties of non-linear electrostatic structure in anisotropic pressure plasma and found that only the width of the soliton depends on the perpendicular pressure; however, an increase in parallel pressure decreases both the amplitude and the width of the soliton. Adnan et al. [31] studied the properties of arbitrary amplitude solitary excitations using Sagdeev's potential approach in an anisotropic superthermal plasma under oblique propagation, and found that the amplitude of solitary pulses increases significantly with a

decrease in the superthermal parameter, which makes the solitary waves steeper and more localized; also, they concluded that the properties of solitary waves are more sensitive to the parallel ion pressure component. Manesh et al. [32] studied the properties of solitary waves in an anisotropic plasma with lighter and heavier ions and found that the light ion's pressure anisotropy determines the polarity of solitary waves, and it is rarefactive for anisotropic lighter ions, whereas it is compressive for isotropic lighter ions. Adnan et al. [33] studied the properties of linear and non-linear dust ion acoustic waves in an anisotropic plasma using the Z-K equation, and reported the existence of compressive and rarefactive solitons in such plasma in the presence of nonthermal electrons; they also found that the compressive and rarefactive solitons propagate with higher phase velocities due to the presence of the electron nonthermal and ion pressure anisotropy. Adnan et al. [26] derived the Z-K equation to study the non-linear electrostatic waves in superthermal electron-positron plasma in the presence of ion pressure anisotropy. It was concluded in their report that the solitary wave's speed is controlled by the superthermal parameter and parallel pressure anisotropy, while perpendicular pressure anisotropy has no role in phase velocity. Khan et al. [34] studied the properties of soliton and cnoidal waves in anisotropic superthermal electron-positron-ion plasma and found that the wavelength of the cnoidal wave structure is reduced upon increasing the parallel and perpendicular anisotropy of ions. Khalid and Rahman [35] studied the ion pressure anisotropy of the ion acoustic non-linear periodic waves in a magnetized plasma. They reported that the increase in parallel pressure of ions decreases the amplitude and width of the ion acoustic periodic waves and the ion acoustic waves behave differently than ion acoustic periodic (cnoidal) waves in anisotropic plasmas. Khalid et al. [36] also studied the propagation of ion-acoustic electrostatic waves in a magnetized electron-ion plasma with pressure anisotropy.

Apart from classical plasmas, the effect of pressure anisotropy has been widely investigated in dense quantum magnetized plasmas. For example, very recently, Bordbar and Karami studied the structural properties of an anisotropic dense neutron star and studied the compactness, redshift, etc., of such dense matter as a function of strong magnetic field of the order of $\sim 10^{17}$ Gauss which creates the anisotropy [37]. Patidar and Sharma [38] studied the MHD wave modes in anisotropic relativistic degenerate plasma and found fast and slow wave modes propagating under the combined influence of various forces such as pressure anisotropy, exchange potential, Bohm force and magnetic field. Irfan et al. [39] observed a strong modification of amplitude and width of weakly non-linear ion acoustic waves considering the pressure anisotropy of positive ions and electron trapping effects in a dense quantum magnetoplasma. Moreover, in the non-relativistic and ultra-relativistic regimes, the anisotropic ion pressure also affects the stability of solitary waves considerably. On the other hand, spin is another intrinsic property which is also a quantum effect, and it has a significant role in the strongly magnetized plasmas. The spin effect can collectively affect the propagation of waves in the laboratory as well as the astrophysical plasma environment [40]. The hydrodynamic model has also been applied to separate the treatment of up-spin and down-spin electron density. The difference in Fermi pressure due to the difference in the concentrations of up-spin and down-spin electrons is important in different realistic plasma situations where the strong magnetic field can create ionic pressure anisotropy [41]. The appearance of new modes has also been reported by the relative contribution of two types of spin electron contributions [42]. The spin effect helps measure the plasma density more accurately and can also be used to measure the magnetic field of the plasma waves [43]. Ahmed et al. [44] studied the relative density effects of spin-up and spin-down degenerate electrons using quantum hydrodynamics and concluded that the phase velocity is influenced by the orientation of the spin in the linear regime, while in the non-linear regime, the amplitude and width become modified due

to spin effect. By considering the spin effect of electrons, numerous investigations have been reported. Sahu et al. [45] applied the quantum hydrodynamic model to study the effect of quantum statistical and Bohm potential terms in magneto-rotating plasma and found that the non-linear properties are strongly affected by the rotation speed. Brodin and Marklund [15] investigated the MHD limit for a pair of plasmas. Considering the MHD equation of degenerate electrons and positrons along with their spin, they showed that microscopic properties like the spin of the electrons and positrons can lead to macroscopic and collective effects in strongly magnetized plasmas. They concluded that the solitary structures vanish, making a true quantum soliton, if the quantum spin effects are neglected. Mushtaq and Vladimirov [46] investigated the linear and non-linear compressional magnetosonic waves in magnetized degenerate spin-1/2 Fermi plasmas and concluded that this effect can significantly modify the amplitude and bandwidth of rarefactive magnetosonic solitons. Kumar and Ahmed [47] investigated filamentation instability of an intense laser beam plasma through the effects of spin polarization caused by a difference in the concentration of oppositely spinning electrons and concluded that the spin polarization enhances the filamentation. Pradhan et al. [48] studied the propagation of small-amplitude quantum ion-acoustic waves and their fractal representation in an electron–ion quantum plasma with separated spin electrons. They found that the spin density polarization ratio has an effect on the amplitude of the wave, while the frequency ratio has no effect. The objective of this work is to study the shock wave propagation in quantum magnetoplasma with the relative contribution of spin-up and spin-down electron concentration, considering the ionic pressure anisotropy of positive ion and ion beams as well as their anisotropic viscosities. The Z-K-B equation is derived using the reductive perturbation technique to study the shock wave nature in such plasma. These plasmas are believed to exist in pulsar magnetosphere, in neutron stars, in active nuclei and in the early universe. The results obtained here may be useful for laboratory as well as space astrophysical plasma environments wherein such plasma environments are prevalent. The manuscript is arranged as follows: Section 2 contains the detailed theoretical formulation, Section 3 contains methodologies as well as detailed derivations of the Z-K-Burger equation, Section 4 contains the results and discussion part, and the overall conclusion is presented in Section 5.

2. Theoretical Formulation

We consider a plasma composed of degenerate electrons with relative contributions of spin-up and spin-down electrons, classical anisotropic and viscous positive ions and positive ion beams. We assume that ion beams have some initial streaming velocity normalized to ion Fermi sound velocity which is equal to V_{b0} . The propagation of shock waves in a spin-polarized degenerate quantum magnetoplasma is studied in the following with the help of Equations (1)–(9). The normalized set of governing equations is given by the following [1,4,27,28,43,44].

$$\frac{\partial N_i}{\partial T} + \frac{\partial(N_{i,b}V_{i,bx})}{\partial x} + \frac{\partial(N_{i,b}V_{i,by})}{\partial y} + \frac{\partial(N_{i,b}V_{i,bz})}{\partial z} = 0 \quad (1)$$

$$\frac{\partial V_{ix}}{\partial T} + \left(V_{ix} \frac{\partial}{\partial x} + V_{iy} \frac{\partial}{\partial y} + V_{iz} \frac{\partial}{\partial z} \right) V_{ix} = -\frac{\partial \phi}{\partial x} + \eta'_{i\parallel} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V_{ix} - \bar{P}_{i\parallel} N_i \frac{\partial N_i}{\partial x}, \quad (2)$$

$$\frac{\partial V_{iy}}{\partial T} + \left(V_{ix} \frac{\partial}{\partial x} + V_{iy} \frac{\partial}{\partial y} + V_{iz} \frac{\partial}{\partial z} \right) V_{iy} = -\frac{\partial \phi}{\partial y} + \eta'_{i\perp} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V_{iy} - \bar{P}_{i\perp} \frac{1}{N_i} \frac{\partial N_i}{\partial y} + V_{iz} \Omega_i, \quad (3)$$

$$\frac{\partial V_{iz}}{\partial T} + \left(V_{ix} \frac{\partial}{\partial x} + V_{iy} \frac{\partial}{\partial y} + V_{iz} \frac{\partial}{\partial z} \right) V_{iz} = -\frac{\partial \phi}{\partial z} + \eta'_{i\perp} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V_{iz} - \bar{P}_{i\perp} \frac{1}{N_i} \frac{\partial N_i}{\partial z} - V_{iy} \Omega_i, \quad (4)$$

$$\frac{\partial V_{bx}}{\partial T} + \left(V_{bx} \frac{\partial}{\partial x} + V_{by} \frac{\partial}{\partial y} + V_{bz} \frac{\partial}{\partial z} \right) V_{bx} = -\gamma \frac{\partial \phi}{\partial x} + \eta'_{b\parallel} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V_{bx} - \bar{P}_{b\parallel} N_b \frac{\partial N_b}{\partial x}, \quad (5)$$

$$\frac{\partial V_{by}}{\partial T} + \left(V_{bx} \frac{\partial}{\partial x} + V_{by} \frac{\partial}{\partial y} + V_{bz} \frac{\partial}{\partial z} \right) V_{by} = -\gamma \frac{\partial \phi}{\partial y} + \eta'_{b\perp} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V_{by} - \bar{P}_{b\perp} \frac{1}{N_b} \frac{\partial N_b}{\partial y} + V_{bz} \Omega_b, \quad (6)$$

$$\frac{\partial V_{bz}}{\partial T} + \left(V_{bx} \frac{\partial}{\partial x} + V_{by} \frac{\partial}{\partial y} + V_{bz} \frac{\partial}{\partial z} \right) V_{bz} = -\gamma \frac{\partial \phi}{\partial z} + \eta'_{b\perp} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V_{bz} - \bar{P}_{b\perp} \frac{1}{N_b} \frac{\partial N_b}{\partial z} - V_{by} \Omega_b, \quad (7)$$

$$\nabla \phi - \frac{(2\delta_{\uparrow\downarrow})^{2/3}}{3} (N_{j\uparrow\downarrow})^{-1/3} \nabla N_{e\uparrow} + \frac{H^2}{2} \nabla \left(\frac{1}{\sqrt{N_{j\uparrow\downarrow}}} \nabla^2 \sqrt{N_{j\uparrow\downarrow}} \right) = 0, \quad (8)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = (N_{e\uparrow} v_{\uparrow} + N_{e\downarrow} v_{\downarrow} - Z_i N_i - Z_b N_b \mu_b), \quad (9)$$

where $\Omega_{i,b} = \omega_{ci,b}/\omega_{ph}$, $\delta_{\uparrow\downarrow} = n_{0\uparrow\downarrow}/n_0$, $T_F = (3\pi^2 n_0)^{2/3} \hbar^2 / 2K_B m_e$, $H = \hbar \omega_p / C_s \sqrt{m_i m_e}$, $\mu_b = n_{b0}/n_{i0}$.

Here, H is the quantum diffraction parameter. Also, the density polarization ratio of spin-up and spin-down electrons is taken as $\kappa = (n_{e0\uparrow} - n_{e0\downarrow}) / (n_{e0\uparrow} + n_{e0\downarrow})$, so accordingly, $\delta_{\uparrow\downarrow} = (1 \pm \kappa)/2$, and also $v_{\uparrow\downarrow} = (1 + \mu_b)(1 \pm \kappa)/2$. Further, $N_{i(b)}$ is the normalized plasma (beam) ion density, $V_{i(b)}$ is the normalized plasma (beam) ion fluid velocity, ϕ is the normalized electrostatic potential, $\eta'_{i(b)}$ is the normalized viscosity for plasma (beam) ion, and $\omega_{ci(b)}$ is the cyclotron frequency of plasma (beam) ion.

The other plasma parameters are normalized as follows: $\varphi = \varepsilon_{Fe} \phi / e$, $t = T \omega_p^{-1}$, $x = X \times \lambda_{Fe}$, $N_j = n_j / n_{j0}$, $\lambda_{Fe} = C_s / \omega_p$, $V_j = v_j / C_s$, $\omega_j = (4\pi n_{i0} e^2 / m_i)^{1/2}$, $\gamma = Z_b m_i / Z_i m_b$, $\lambda_{Fe} = (\varepsilon_{Fe} / 4\pi n_{i0} e^2)^{1/2}$, $C_s = \sqrt{\varepsilon_{Fe} / m_i}$, $P_{i(b)\parallel} = P_{i(b)\parallel 0} \left(\frac{n_{i(b)}}{n_{i(b)0}} \right)^3$, $P_{i(b)\perp} = P_{i(b)\perp 0} \left(\frac{n_{i(b)}}{n_{i(b)0}} \right)$, $\varepsilon_{Fe} = (3\pi^2 n_{e\uparrow\downarrow})^{2/3} \hbar^2 / 2m_{e\uparrow\downarrow}$, where λ_{Fe} is the Thomas–Fermi length, C_s is the Fermi ion sound velocity, ω_{ph} is the plasma frequency, $Z_{i(b)}$ is the charged state of plasma (beam) ion, $m_{i(b)}$ is the mass of plasma (beam) ion, and $n_{e\uparrow\downarrow}$, n_i , n_b are the electron (up, down) ion and beam densities, respectively. $P_{i(b)\parallel}$ and $P_{i(b)\perp}$ are the parallel and perpendicular pressure anisotropy of plasma (beam) ion. $p_{Fe\uparrow\downarrow} = (m_e v_{Fe\uparrow\downarrow}^2 / 5 n_{e0\uparrow\downarrow}^{2/3}) n_{e\uparrow\downarrow}^{5/3}$ is the Fermi pressure for the up (down)-spin electron, and $v_{Fe\uparrow\downarrow}^2 = \hbar^2 (3\pi^2 n_{e0\uparrow\downarrow})^{2/3} / m_e^2$ is the Fermi speed of the up (down)-spin electrons with $\hbar = h / 2\pi$, h being Planck's constant. The pressure equations for the anisotropic and adiabatic system are given by Chew–Goldberger–Low popularly known as (CGL) or double adiabatic theory [49,50], according to which $\frac{d}{dt} \left(\frac{\bar{P}_{i\parallel}}{n_i^3} B^2 \right) = 0$ and $\frac{d}{dt} \left(\frac{\bar{P}_{i\perp}}{n_i B} \right) = 0$. In the case of electrostatic waves in a magnetized plasma, the ambient magnetic field $B = (B_0)$ is constant with time, i.e., $\frac{d}{dt} (B) = 0$, where B_0 is the magnetic field at equilibrium. Moreover, the normalized parallel and perpendicular ion pressures obtained from the CGL theory are given as $\bar{P}_{i\parallel} = 3P_{i\parallel 0} / n_{i0} \varepsilon_{Fe}$, $\bar{P}_{i\perp} = P_{i\perp 0} / n_{i0} \varepsilon_{Fe}$, where $P_{i\parallel 0} = n_{i0} T_{i\parallel}$ and $P_{i\perp 0} = n_{i0} T_{i\perp}$ are the equilibrium values of parallel and perpendicular pressure functions, respectively, and n_{i0} is the unperturbed ion density [34,49,51]. On the other hand, the variations in the ambient magnetic field alter the ionic temperatures in parallel and perpendicular directions to the magnetic field, i.e., $T_{i\parallel} \propto B_0$ and $T_{i\perp} \propto \left(\frac{n_{i0}}{B_0} \right)^2$, respectively [50,51].

3. Derivation of Z-K-Burger Equation

Here, we used the reductive perturbation technique to derive the evolution equation of the shock wave. The stretched coordinates [1,4,28,46] are given by

$$\xi = \varepsilon^{\frac{1}{2}}(X - M\tau), \quad \eta = \varepsilon^{\frac{1}{2}}Y, \quad \zeta = \varepsilon^{\frac{1}{2}}Z, \quad T = \varepsilon^{\frac{3}{2}}\tau, \quad \eta'_{i,b\parallel} = \varepsilon^{1/2}\eta'_{i,b\parallel 0}, \quad \eta'_{i,b\perp} = \varepsilon^{1/2}\eta'_{i,b\perp 0} \quad (10)$$

where M is the phase velocity (Mach number), and ε is a small nonzero constant measuring the strength of dispersion. The physical variables in equations are expanded in a power series in terms of the expansion parameter ε as

$$\left. \begin{aligned} N_{j\uparrow\downarrow} &= 1 + \varepsilon N_{j\uparrow\downarrow}^{(1)} + \varepsilon^2 N_{j\uparrow\downarrow}^{(2)} + \varepsilon^3 N_{j\uparrow\downarrow}^{(3)} + \dots \\ V_{ix} &= 0 + \varepsilon V_{ix}^{(1)} + \varepsilon^2 V_{ix}^{(2)} + \varepsilon^3 V_{ix}^{(3)} + \dots \\ V_{bx} &= V_{b0} + \varepsilon V_{bx}^{(1)} + \varepsilon^2 V_{bx}^{(2)} + \varepsilon^3 V_{bx}^{(3)} + \dots \\ V_{jy,z} &= \varepsilon^{\frac{3}{2}} V_{jy,z}^{(1)} + \varepsilon^2 V_{jy,z}^{(2)} + \varepsilon^{\frac{5}{2}} V_{jy,z}^{(3)} + \dots \\ V_{by,z} &= \varepsilon^{\frac{3}{2}} V_{by,z}^{(1)} + \varepsilon^2 V_{by,z}^{(2)} + \varepsilon^{\frac{5}{2}} V_{by,z}^{(3)} + \dots \\ \phi &= \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \varepsilon^3 \phi^{(3)} + \dots \end{aligned} \right\} \quad (11)$$

Substituting the above stretched coordinates from Equation (10) and the respective expansions from Equation (11) in Equations (1)–(9) and then collecting the terms appearing in the lowest order of ε give the following relations:

$$\begin{aligned} N_i^{(1)} &= \frac{-\phi^{(1)}}{\bar{P}_{i\parallel} - M^2}, \quad N_b^{(1)} = \frac{-\gamma\phi^{(1)}}{\bar{P}_{b\parallel} - (M - V_{b0})^2}, \quad V_{ix}^{(1)} = \frac{-M}{\bar{P}_{i\parallel} - M^2} \phi^{(1)} \\ V_{bx}^{(1)} &= \frac{-(M - V_{b0})\gamma}{\bar{P}_{b\parallel} - (M - V_{b0})^2} \phi^{(1)}, \quad N_{e\uparrow}^{(1)} = \frac{3\phi^{(1)}}{(2\delta_{\uparrow})^{2/3}}, \quad N_{e\downarrow}^{(1)} = \frac{3\phi^{(1)}}{(2\delta_{\downarrow})^{2/3}} \end{aligned} \quad (12)$$

$$v_{\uparrow} N_{e\uparrow}^{(1)} + v_{\downarrow} N_{e\downarrow}^{(1)} = Z_i N_i^{(1)} + Z_b \mu_b N_b^{(1)} \quad (13)$$

Using Equation (12) into Equation (13), we obtain the phase velocity as

$$M^2 = \pm \sqrt{\left(\frac{-Z_i (\bar{P}_{b\parallel} - (M - V_{b0})^2) - Z_b \mu_b \gamma (\bar{P}_{i\parallel} - M^2)}{\left(\frac{\bar{P}_{i\parallel}}{M^2} - 1 \right) \left(\bar{P}_{b\parallel} - (1 - \frac{V_{b0}}{M})^2 \right) \left(v_{\uparrow} \left(\frac{3}{(2\delta_{\uparrow})^{2/3}} \right) + v_{\downarrow} \left(\frac{3}{(2\delta_{\downarrow})^{2/3}} \right) \right)} \right)} \quad (14)$$

Similarly, the higher order of ε from Equations (1)–(9) with second-order quantities gives the following equations:

$$\begin{aligned} V_{iy}^{(2)} &= \frac{M}{\Omega_i^2} \frac{\partial}{\partial \xi} \left(\frac{\partial \phi^{(1)}}{\partial \eta} + \bar{P}_{i\perp} \frac{\partial N_i^{(1)}}{\partial \eta} \right), \quad V_{iz}^{(2)} = \frac{M}{\Omega_i^2} \frac{\partial}{\partial \xi} \left(\frac{\partial \phi^{(1)}}{\partial \xi} + \bar{P}_{i\perp} \frac{\partial N_i^{(1)}}{\partial \xi} \right) \\ V_{by}^{(2)} &= \frac{M - V_{b0}}{\Omega_b^2} \frac{\partial}{\partial \xi} \left(\gamma \frac{\partial \phi^{(1)}}{\partial \eta} + \bar{P}_{b\perp} \frac{\partial N_b^{(1)}}{\partial \eta} \right), \quad V_{bz}^{(2)} = \frac{M - V_{b0}}{\Omega_b^2} \frac{\partial}{\partial \xi} \left(\gamma \frac{\partial \phi^{(1)}}{\partial \xi} + \bar{P}_{b\perp} \frac{\partial N_b^{(1)}}{\partial \xi} \right) \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{\partial N_i^{(2)}}{\partial \xi} &= \frac{2M}{(\bar{P}_{i\parallel} - M^2)^2} \frac{\partial \phi^{(1)}}{\partial \tau} - \frac{3M^2 + \bar{P}_{i\parallel}}{(\bar{P}_{i\parallel} - M^2)^3} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} - \left(\frac{M^2}{\Omega_i^2 (\bar{P}_{i\parallel} - M^2)} - \frac{M^2 \bar{P}_{i\perp}}{\Omega_i^2 (\bar{P}_{i\parallel} - M^2)^2} \right) \frac{\partial}{\partial \xi} \left(\frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} \right) \\ &\quad - \frac{M \eta'_{i\parallel 0}}{(\bar{P}_{i\parallel} - M^2)^2} \left(\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} \right) \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{\partial N_b^{(2)}}{\partial \xi} &= \frac{2(M - V_{b0})\gamma}{(\bar{P}_{b\parallel} - (M - V_{b0})^2)^2} \frac{\partial \phi^{(1)}}{\partial \tau} - \frac{3(M - V_{b0})^2 \gamma^2 + \bar{P}_{b\parallel} \gamma^2}{(\bar{P}_{b\parallel} - (M - V_{b0})^2)^3} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} \\ &\quad - \left(\frac{(M - V_{b0})^2 \gamma}{\Omega_b^2 (\bar{P}_{b\parallel} - (M - V_{b0})^2)} - \frac{(M - V_{b0})^2 \bar{P}_{b\perp} \gamma}{\Omega_b^2 (\bar{P}_{b\parallel} - (M - V_{b0})^2)^2} \right) \frac{\partial}{\partial \xi} \left(\frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} \right) - \frac{\eta'_{b\parallel 0} (M - V_{b0}) \gamma}{(\bar{P}_{b\parallel} - (M - V_{b0})^2)^2} \left(\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} \right) \end{aligned} \quad (17)$$

$$\frac{\partial N_{e\uparrow}^{(2)}}{\partial \xi} = \frac{3}{(2\delta_{\uparrow})^{4/3}} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{H^2}{4} \left(\frac{9}{(2\delta_{\uparrow})^{4/3}} \right) \frac{\partial}{\partial \xi} \left(\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} \right) \quad (18)$$

$$\frac{\partial N_{e\downarrow}^{(2)}}{\partial \xi} = \frac{3}{(2\delta_{\downarrow})^{4/3}} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{H^2}{4} \left(\frac{9}{(2\delta_{\downarrow})^{4/3}} \right) \frac{\partial}{\partial \xi} \left(\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} \right) \quad (19)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = v_{\uparrow} N_{e\uparrow}^{(2)} + v_{\downarrow} N_{e\downarrow}^{(2)} - Z_i N_i^{(2)} - Z_b \mu_b N_b^{(2)} \quad (20)$$

Solving Equations (15)–(20) along with Equation (12), we have the equation:

$$q \frac{\partial \phi^{(1)}}{\partial \tau} + p \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + (1+r) \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + (r+s) \frac{\partial}{\partial \xi} \left(\frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} \right) - t \left(\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} \right) = 0 \quad (21)$$

where

$$\begin{aligned} p &= \left(-\frac{3v_{\uparrow}}{(2\delta_{\uparrow})^{4/3}} - \frac{3v_{\downarrow}}{(2\delta_{\downarrow})^{4/3}} - \frac{Z_i(3M^2 + \bar{P}_{i\parallel})}{(\bar{P}_{i\parallel} - M^2)^3} - \frac{Z_b \mu_b \gamma^2 (3(M - V_{b0})^2 + \bar{P}_{b\parallel})}{(\bar{P}_{b\parallel} - (M - V_{b0})^2)^3} \right) \\ q &= \left(\frac{2Z_i M}{(\bar{P}_{i\parallel} - M^2)^2} + \frac{2Z_b \mu_b \gamma (M - V_{b0})}{(\bar{P}_{b\parallel} - (M - V_{b0})^2)^2} \right) \\ r &= \left(-\frac{H^2}{4} \left(\frac{9v_{\uparrow}}{(2\delta_{\uparrow})^{4/3}} \right) - \frac{H^2}{4} \left(\frac{9v_{\downarrow}}{(2\delta_{\downarrow})^{4/3}} \right) \right) \\ s &= \left(\left(-\frac{Z_i M^2}{\Omega_i^2 (\bar{P}_{i\parallel} - M^2)} + \frac{Z_i M^2 \bar{P}_{i\perp}}{\Omega_i^2 (\bar{P}_{i\parallel} - M^2)^2} \right) + \left(-\frac{Z_b \mu_b (M - V_{b0})^2 \gamma}{\Omega_b^2 (\bar{P}_{b\parallel} - (M - V_{b0})^2)} + \frac{Z_b \mu_b (M - V_{b0})^2 \bar{P}_{b\perp} \gamma}{\Omega_b^2 (\bar{P}_{b\parallel} - (M - V_{b0})^2)^2} \right) \right) \\ t &= \left(\frac{Z_i M \eta'_{i\parallel 0}}{(\bar{P}_{i\parallel} - M^2)^2} + \frac{\eta'_{b\parallel 0} Z_b \mu_b \gamma (M - V_{b0})}{(\bar{P}_{b\parallel} - (M - V_{b0})^2)^2} \right) \end{aligned} \quad (22)$$

Finally, we derive the Zakharov–Kuznetsov–Burgers equation in the form:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + (B + C) \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + (C + D) \frac{\partial}{\partial \xi} \left(\frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} \right) - E \left(\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} \right) = 0 \quad (23)$$

where $A = \frac{p}{q}$, $B = \frac{1}{q}$, $C = \frac{r}{q}$, $D = \frac{s}{q}$, $E = \frac{t}{q}$.

To obtain the solution of Equation (22), we introduce the new variable $\chi = (l_x \xi + l_y \eta + l_z \zeta - U\tau)$, where χ is the transformed coordinate with respect to a frame moving with velocity U . l_x, l_y, l_z are the direction cosines along ξ, η, ζ , respectively, such that $l_x^2 + l_y^2 + l_z^2 = 1$, and then, taking $\phi^{(1)} = \phi$, Equation (22) becomes

$$-\frac{U}{Al_x} \frac{d\phi}{d\chi} + \phi \frac{d\phi}{d\chi} + F \frac{d^3 \phi}{d\chi^3} - G \frac{d^2 \phi}{d\chi^2} = 0 \quad (24)$$

Here, $F = \frac{1}{A} (Bl_x^2 + C + D(l_y^2 + l_z^2))$, $G = \frac{E}{Al_x}$.

Now, employing the *Tanh* [52] method in Equation (23), we obtain the shock wave solution as

$$\phi = \frac{12}{25} \frac{G^2}{F} \left[1 - \frac{1}{4} \left(1 + \tanh \left(\frac{G}{10F} \chi \right) \right)^2 \right] \quad (25)$$

4. Results and Discussion

From the analytical solution governed by Equation (24), we analyzed the nature and characteristics of shock wave propagation under different physical situations. The data range adopted in the present manuscript is well established in different plasma environments. For example, we considered the plasma density of the order of $10^{26} - 10^{30} \text{ cm}^{-3}$ and magnetic field of the order of $10^{10} - 10^{12} \text{ G}$ with $T_e \sim 1 \text{ keV}$, which is a well-known plasma environment of different astrophysical plasma origins such as white dwarfs, neutron stars, etc., [1–10,24–26,41,53,54] as well as in some high-density laser plasma laboratory experiment where the ion beams are considered as an indispensable tool [16,55–61] to create a plasma environment of the origin of a white dwarf, neutron star, pulsar, magnetars, etc.

Figure 1a,b describes the propagation of normalized phase velocities, i.e., the Mach number of the ion beam plasma system as a function of the normalized beam velocity under different values of parallel pressures of ion beam and positive ions. As evident from the figure and also reported earlier [1,4,62,63], in an isotropic, classical as well as quantum plasma, three modes appear when ion beams with finite velocity are injected into the plasma system, namely, fast beam mode, slow beam mode and an inherent ion acoustic mode apart from a coupled mode which appears under the class of ion–ion instability, depending on the streaming speed of ion beams. The phase velocity of ion acoustic waves supported by beam ions are shifted by beam velocity, and as a result, we obtain two beam modes, fast and slow, whereas the background ion supports the inherent ion acoustic mode. In classical plasmas, the coupled mode, i.e., the typical ion–ion instability appears when the beam velocity is less than twice the ion sound velocity, while with beam velocity $V_b > 2C_s$, the abovementioned three modes appear. Here, it is to be noted that the distinction between different modes is made based on the nature of appearance. For example, in classical or even in quantum isotropic plasmas, the mode which propagates with finite growth rate separately appears as fast mode, and the mode which appears at $1 \leq V_b/C_s \leq 2$ is called the coupled mode, as the ion and ion beams are strongly coupled which falls under ion–ion instability. Then, this coupled mode further bifurcates at $V_b/C_s \geq 2$ into an ion acoustic mode (which is being unaffected even with increasing beam velocity) and a slow beam mode (phase velocity increases with increasing beam velocity) [62]. However, here, due to the presence of ionic pressure anisotropy as well as different spin density polarization ratios of up-spin and down-spin electrons, apart from the modes that appear in classical as well as quantum plasmas, a completely new mode appears at the onset of the fast beam mode whose variation with beam velocity is very small (red thick curve in inset of Figures 1–3). Since the rate of variation of this mode with beam velocity is very small compared to the other modes of ion beam plasma, we have coined this mode as the stable beam mode. However, the nature of the appearance of the other modes slightly differs from previous observations though their characteristics are more or less similar [1,4,62,63]. For example, the coupled mode starts appearing nearly at $V_b/C_s \approx 1.2$ (Figure 1a), and before that, the ion acoustic mode (blue large dashed curve) and the slow beam mode (black dot dashed curve) appear. In this region, the slow beam mode remains relatively unaffected, while the normalized phase velocity of the ion acoustic mode initially decreases and then increases up to $V_b/C_s \approx 1.2$. However, the range of beam velocity at which the coupled mode appears is different for different physical situations, e.g., it is $V_b/C_s \approx 2$ (Figure 3b), whereas it is $V_b/C_s \approx 1.4$ (Figure 3a), though their characteristics are the same. Interestingly, the behaviour of the slow beam mode and ion acoustic mode that appears beyond $V_b/C_s \geq 2$ is similar to the isotropic classical or quantum plasmas, though the behaviour of slow beam mode and the ion acoustic mode at $V_b/C_s \leq 2$ (basically in between 1.2 and 1.4 of Figures 1 and 2) is quite different. In fact, ion acoustic and slow mode does not appear in classical or in quantum plasmas without anisotropic effect and spin effect for $V_b/C_s \leq 2$,

see Refs. [1,4,62–64]. Moreover, in an anisotropic spin-polarized quantum magnetoplasma, it is clear from Figures 1–3 that the range of beam velocities for the appearance of different modes is greatly modified. For example, if we consider Figure 3a, we see that the typical ion–ion instability (coupled mode) starts at nearly $V_b/C_s \approx 1.5$ which is greater than the value of V_b/C_s at which the coupled mode starts in the case of Figures 1 and 2. This is because, in Figure 3a, we increased the parallel pressures of both the ion and ion beams simultaneously keeping the other parameter constant compared to Figures 1 and 2. On the other hand, if we closely observe Figure 3b, we see that the coupled mode starts at $V_b/C_s \approx 2$, which is higher than all the cases considered from Figures 1–3. Moreover, we find that the typical stable mode becomes supersonic for the physical situation considered in the case of Figure 3b, whereas, for all other physical situation considered from Figures 1a, 2a and 3a, the stable mode is subsonic. This is because in Figure 3b, the parallel pressures of ion and ion beam as well as ion beam density increased simultaneously compared to the physical situation considered from Figures 1a, 2a and 3a. Another important observation is that from Figures 1a and 2a, we have plotted the Mach number for the case when the concentration up-spin electron is more than the down-spin electron ($\kappa = 1$), and from Figures 1b and 2b, we have plotted the Mach number for the case when the concentration up-spin electron is equal to the down-spin electron ($\kappa = 0$), and interestingly, we have noticed that the phase velocity for all the modes increases when the concentration up-spin electron is more than the down-spin electron. We have noticed a similar rate of increase in phase velocity for the cases considered in Figure 3a,b when the concentration of the up-spin electron is more than the down-spin electron, though we have not reported this here. Thus, we can conclude that the Mach number for the ion beam plasma system increases when the concentration of the up-spin electron is more than the down-spin electron as well as when the parallel pressure of the ion and ion beam increases with an increase in the concentration of ion beams. Apart from these, we can also conclude that with the increase in the ionic parallel pressure and ion beam pressure as well as with the increase in ion beam density, the requirement of beam velocity for the appearance of different ion beam plasma modes—coupled mode, slow mode and ion acoustic mode—changes and increases with increase in the value of parallel pressures of ion and ion beam as well as with an increase in the ion beam density.

In Figures 4–6, we plotted the temporal evolution of the Z-K-B equation governed by Equation (22) at different intervals of time. To carry out the numerical simulation, we assumed a shock wave pulse at $\tau = 0$ to be the initial condition, i.e., $\phi(\chi, 0) = \phi_m \left[1 - \frac{1}{4}(1 + \tanh(\chi/W))^2 \right]$ having a potential ϕ in the space domain χ with ϕ_m and W being the maximum amplitude and width of the shock given by $\phi_m = (12/25)(G^2/F)$ and $W = (10F/G)$, respectively, which is eventually the steady state solution of the Z-K-B equation, and carried out the numerical simulation of the third-order non-linear partial differential equation (NPDE) governed by Equation (22) in MATHEMATICA 11.1.1. As seen from the figures, a monotonic shock like structure at $\tau = 0$ develops into an oscillatory shock structure with increasing τ . This can be understood from the fact that when the dispersion of the wave increases compared to dissipation, one can have an oscillatory shock structure, whereas if the dissipation increases compared to dispersion, one can encounter a monotonic shock-like structure. So, at $t = 0$, which means at the onset of the wave when the wave is yet to undergo significant propagation which otherwise means the wave is yet to undergo strong dispersion, the dissipation is stronger, and obviously, the wave appears to be monotonic. However, as time elapses, with higher time intervals, the frequency of the wave decreases, and as we all know, the low-frequency wave can undergo higher dispersion, so obviously, with higher dispersion, the wave appears to be oscillatory. From the numerical evolution, we see from Figure 4a that the fast beam mode is a compressive one, whereas the slow beam mode as depicted in Figure 4b appears as rarefactive for the

same range of chosen plasma parameters. Now, this is an interesting behaviour of ion beam plasma which has been reported in our earlier observation, both theoretically as well as experimentally [1,4,64], that there is always a distinction between the appearance of the fast and slow beam mode, i.e., if the fast beam mode is compressive then the slow beam mode is rarefactive, and vice versa. Thus, the numerical investigation also affirms the earlier theoretical and experimental findings in classical as well as quantum plasmas. Figure 5a,b describe the temporal evolution of the ion acoustic mode for two different beam velocities, i.e., $V_b = 0.2C_s$ and $V_b = 2.2C_s$. The reason behind choosing two velocities is that in the neighbourhood of $V_b = 0.2C_s$, the Mach number of ion acoustic mode decreases and then increases, and in the neighbourhood of $V_b = 2.2C_s$, the Mach number remains stable. As evident from the figure, in the region where the Mach number decreases, the shock potential is a rarefactive one, whereas in the region where the Mach number increases and becomes stable, the shock potential is a compressive one. Normally, in classical and quantum plasmas, the ion acoustic and the slow beam mode appear beyond $V_b = 2C_s$ and before that, i.e., in the region from $V_b = 0$ to $V_b = 2C_s$, the coupled mode dominates. But here, probably due to the consideration of pressure anisotropy of ion and ion beams as well as the spin contribution of degenerate electrons, the ion acoustic and the slow mode appear before $V_b/C_s = 1$. Then, the coupled mode appears at nearly $V_b = 1.2C_s$ which continues to nearly $V_b = 2C_s$, and beyond $V_b = 2C_s$, the coupled mode bifurcates to the slow mode and ion acoustic mode. On the other hand, Figure 6 demonstrates the temporal evolution of the stable beam mode at two different beam velocities $V_b = 0.6C_s$ and $V_b = 2.5C_s$. As evident from the variation in Mach number depicted in Figures 1–3, the Mach number or the normalized phase velocity decreases gradually with increasing beam velocity, which is also confirmed numerically in Figure 6a,b. The shock potential decreases with increasing beam velocity (Figure 6b). This can also be confirmed from the numerical evolution of temporal oscillation of the shock potential of the stable beam mode, which shows that the temporal oscillation reduces with increasing beam velocity from $V_b = 0.6C_s$ (Figure 6a) and $V_b = 2.5C_s$ (Figure 6b).

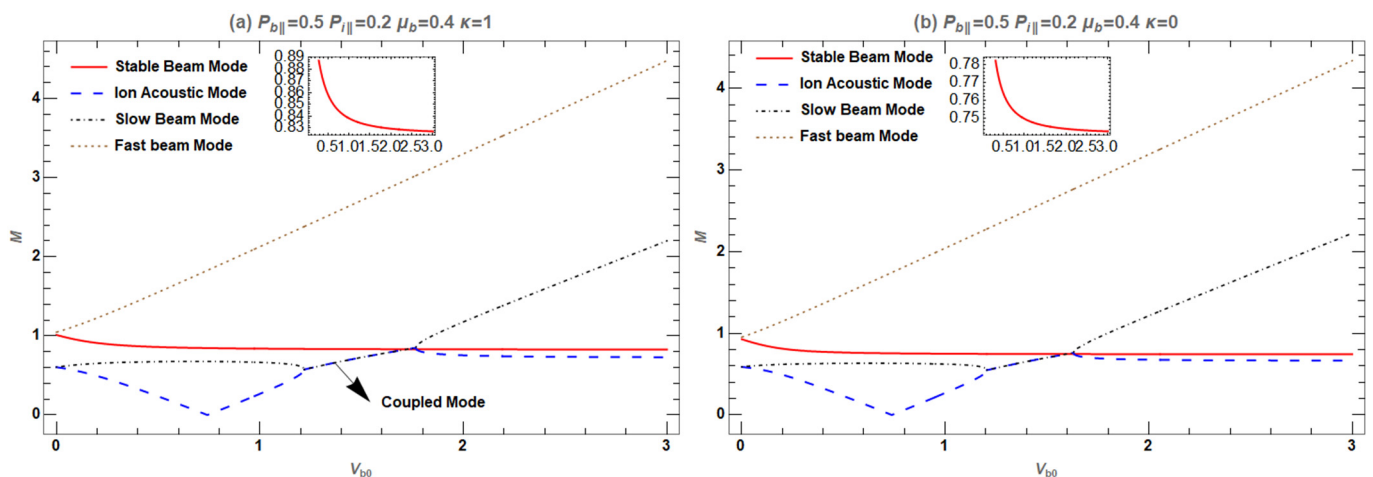


Figure 1. (a) Mach number variation of different ion beam modes with normalized beam velocity when the concentration of spin-up electrons is more than the spin-down electrons with parallel pressure of beam ions higher than the ions. The features of all the modes are sketched. The inset of (a) shows the variation of the stable beam mode with beam velocity. (b) Mach number variation of different ion beam modes with normalized beam velocity when the concentration of spin-up electrons is equal to the spin-down electrons with parallel pressure of beam ions higher than the ions. The inset of (b) shows the variation of the stable beam mode with beam velocity. Here, it is to be noted that $V_{b0} = V_b/C_s$.

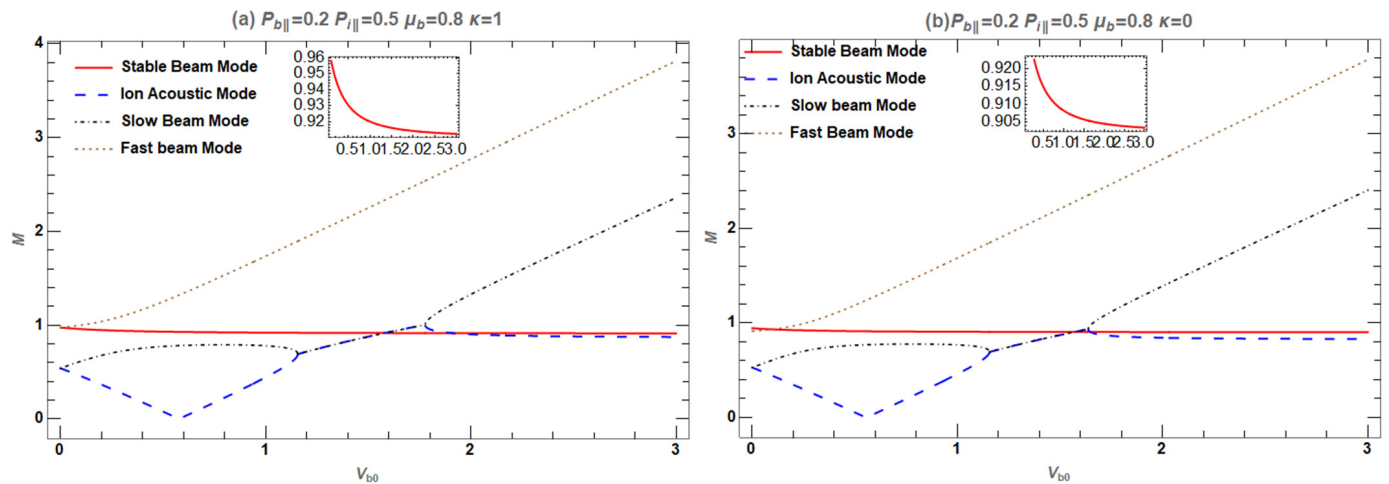


Figure 2. (a) Mach number variation of different ion beam modes with normalized beam velocity when the concentration of spin-up electrons is more than the spin-down electrons with parallel pressure of ions lower than the beam ions. The features of all the modes are sketched. The inset of (a) shows the variation of the stable beam mode with beam velocity. (b) Mach number variation of different ion beam modes with normalized beam velocity when the concentration of spin-up electrons is equal to the spin-down electrons with parallel anisotropy of ions lower than the beam ions. The inset of (b) shows the variation of the stable beam mode with beam velocity. Here, it is to be noted that $V_{b0} = V_b / C_s$.

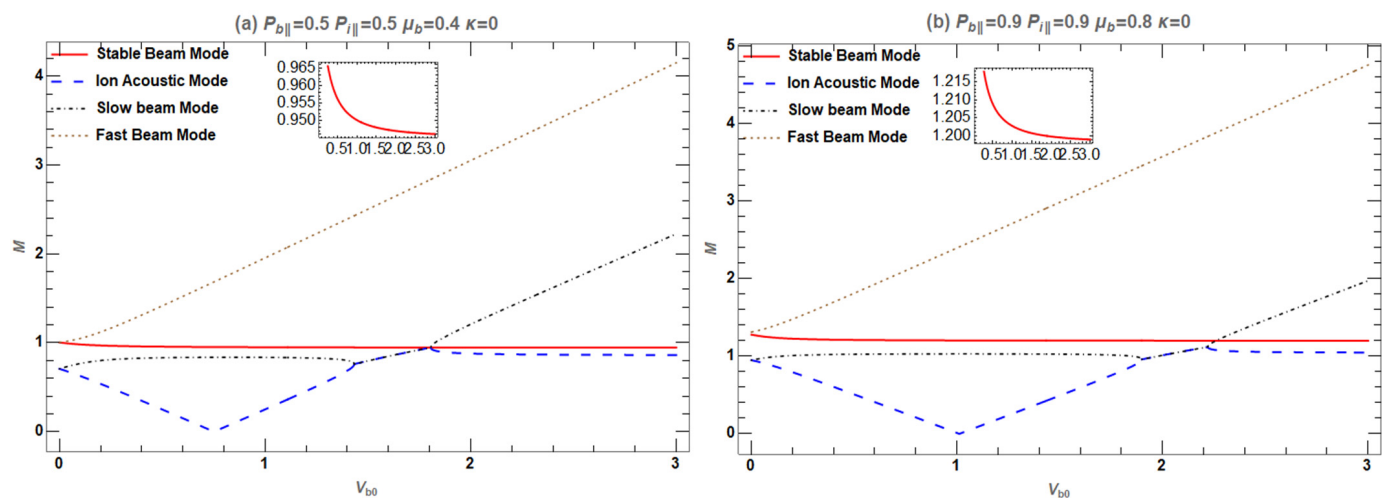


Figure 3. (a) Variation in normalized phase velocity (Mach Number) of different ion beam modes with normalized beam velocity when the concentration of spin-up electrons is more than the spin-down electrons with parallel pressure of ions is equal to the beam ions. The features of all the modes are sketched. The inset of (a) shows the variation in the stable beam mode with beam velocity. (b) Variation in normalized phase velocity (Mach Number) of different ion beam modes with normalized beam velocity when the concentration of spin-up electrons is equal to the spin-down electrons with parallel anisotropy of ions is equal to the beam ions. The inset of (b) shows the variation of the stable beam mode with beam velocity. Here, it is to be noted that $V_{b0} = V_b / C_s$.

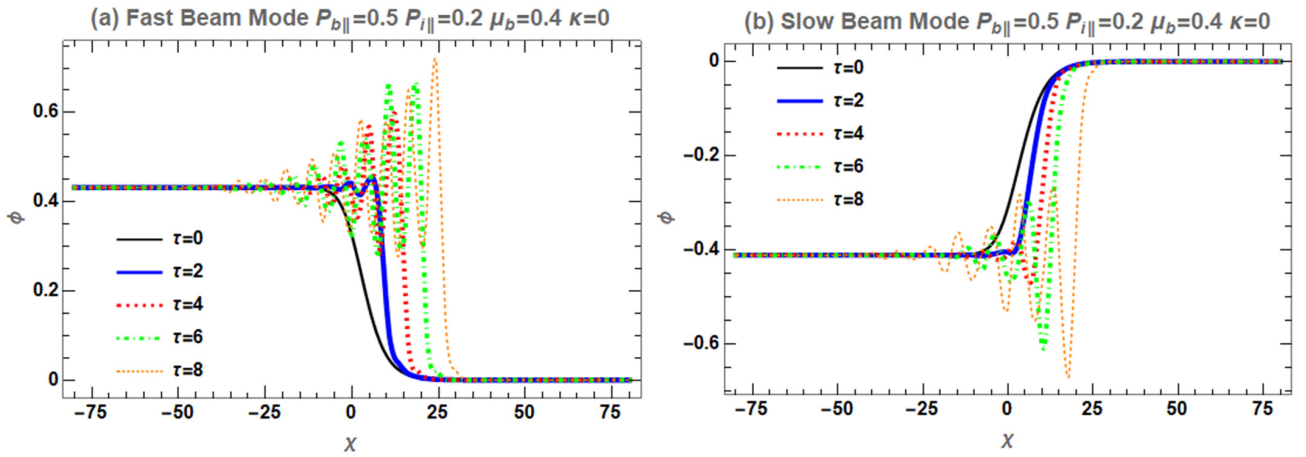


Figure 4. (a) Numerical evolution of shock wave potential profile of fast beam mode with parallel pressure of beam ions higher than the ions for different values of τ . (b) Numerical evolution of shock wave potential profile of slow beam mode with parallel pressure of beam ions higher than the ions for different τ . Here, we have considered $V_b/C_s = 2.5$.

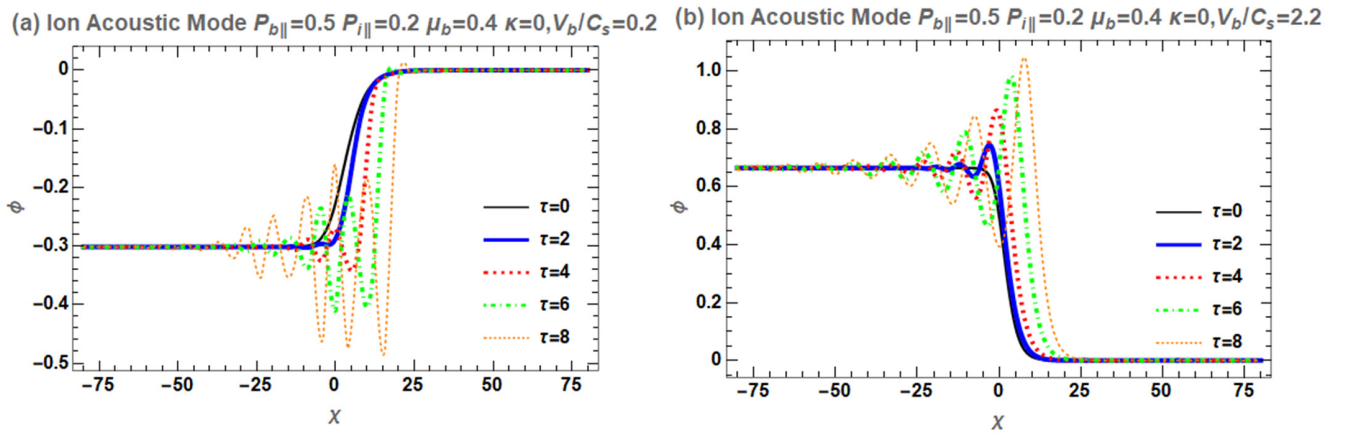


Figure 5. (a) Numerical evolution of shock wave potential profile of ion acoustic mode with parallel pressure of beam ions higher than the ions for different values of τ . (b) Variation in shock wave potential profile of ion acoustic mode with parallel pressure of beam ions higher than the ions for different τ .

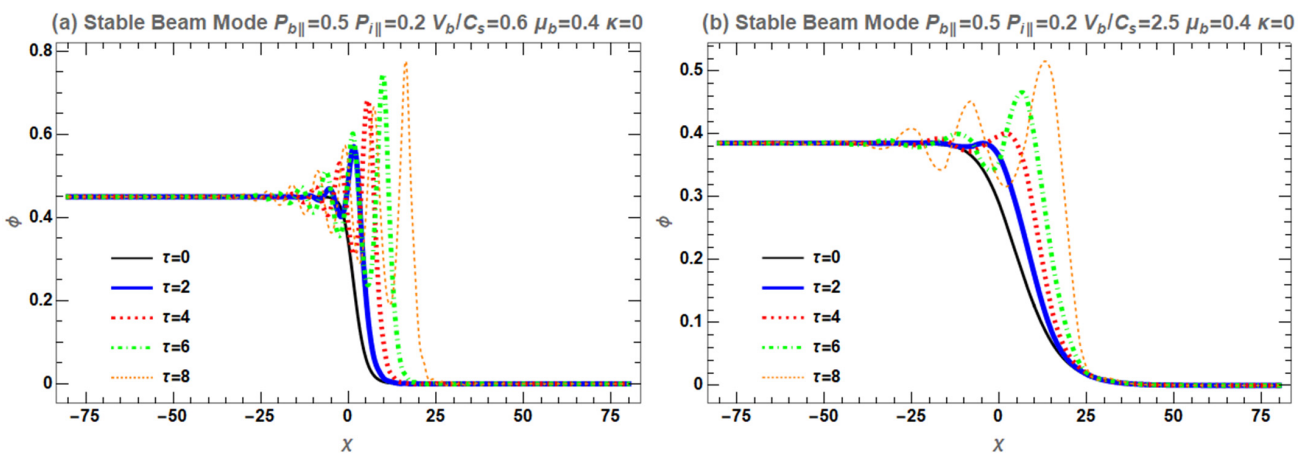


Figure 6. (a) Numerical evolution of shock wave potential profile of stable beam mode with parallel pressure of beam ions higher than the ions for different values of τ . (b) Variation in shock wave potential profile of stable beam mode with parallel pressure of beam ions higher than the ions for different τ .

Figure 7 describes the variation in the steady state shock potential profile of the fast beam mode for the various combinations of perpendicular pressure values of positive ions and ion beams with different values of the viscosity parameter of both the ion and ion beams, for the case when the parallel pressure of ion beams is greater than the positive ions. Figure 7a is plotted for the case of equal concentration of up-spin and down-spin electrons while keeping the other parameters intact, and Figure 7b is plotted for the case when the concentration of up-spin electron is higher than the down-spin electron. It has been observed that the values of perpendicular pressures of ion beam and positive ions as well as the viscosity of positive ion and positive ion beams have a significant role in the maximum amplitude and width of the shock wave pattern. For example, if we consider Figure 7a, it is observed that the under the condition of equal concentration of up-spin and down-spin electrons, the shock potential is found to be the highest (the uppermost brown tiny dash curve) for which the positive ions and beam ions are anisotropic (different values of parallel and perpendicular pressure, $P_{b(i)\perp} \neq P_{b(i)\parallel}$) with beam ions having the highest chosen value of the viscosity parameter. It is primarily because as it is a beam-induced fast mode and it is anisotropic with the highest chosen value of perpendicular pressure, the propagation perpendicular to the magnetic field is highly affected. Moreover, it is well known that dissipative factors like viscosity always favour the shock formation process, and as the beam ions have the highest chosen value of viscosity, it naturally favours the formation shock; thus, we see the beam-induced fast mode propagating with the highest possible shock potential. Apart from this, as the positive ions are the least viscous amongst the chosen value of the viscosity parameter, the interaction of positive ion and ion beams can be expected to be the least, which may further allow the beam-induced fast mode to travel with the highest possible velocity, which in turn results in the highest possible value of the shock potential. On the other hand, if we consider the three lowermost curves, i.e., the blue dashed, the red dot dashed and the orange dotted curve, we find that the shock potential is the least when both the ion and ion beams are anisotropic and the positive ions are more viscous than the beam ions (blue dashed curve); the shock potential is the highest when the positive ions are isotropic but highly viscous (orange dotted curve). This is because since this is a beam-induced fast mode where both the ion and ion beams are anisotropic, there is more interaction of viscous positive ions with beam ions in the direction of propagation which will eventually slow down the fast beam mode, which is exactly the physical situation in case of the blue dashed curve. On the other hand, with positive ions being isotropic, i.e., $P_{i\perp} = P_{i\parallel}$, and strongly viscous (orange dotted curve), the interaction of positive ions with beam ions in the direction of propagation will be the least, which eventually allows the beam-induced fast mode to propagate with higher velocity and hence higher shock potential. Apart from this, we can also conclude that shock wave propagation is better localized (the blue dashed, the red dot dashed and the orange dotted curve) when the positive ions are more viscous than the beam ions (brown tiny dash and green thick dashed curve). Considering the same physical parameters, the nature of shock wave propagation is investigated in Figure 7b when the up-spin electron concentration is more than the down-spin electron. It is seen that the pattern of variation in the shock wave potential is similar to that sketched in the case of equal concentration of up-spin and down-spin electrons, except when the concentration of up-spin electrons is more than the down-spin electrons, the shock wave propagates with a higher potential. This is because we know that the ion acoustic wave propagates as a combination of compression and rarefaction, and due to the higher concentration of spin-up electron density, the electric field arising from the bunching of ions is shielded more effectively by the electrons, thereby preventing the ions from going to the region of rarefaction as the ions normally tend to disperse due to the electric field, and thus, the compression of the wave increases.

This phenomenon of increasing the compression of the ion acoustic wave in the presence of a higher concentration of up-spin electrons than the down-spin electron is explained eloquently in the case of Figures 10 and 11, as the situation demands a detailed analysis to understand the physics underlying the context of Figures 10 and 11.

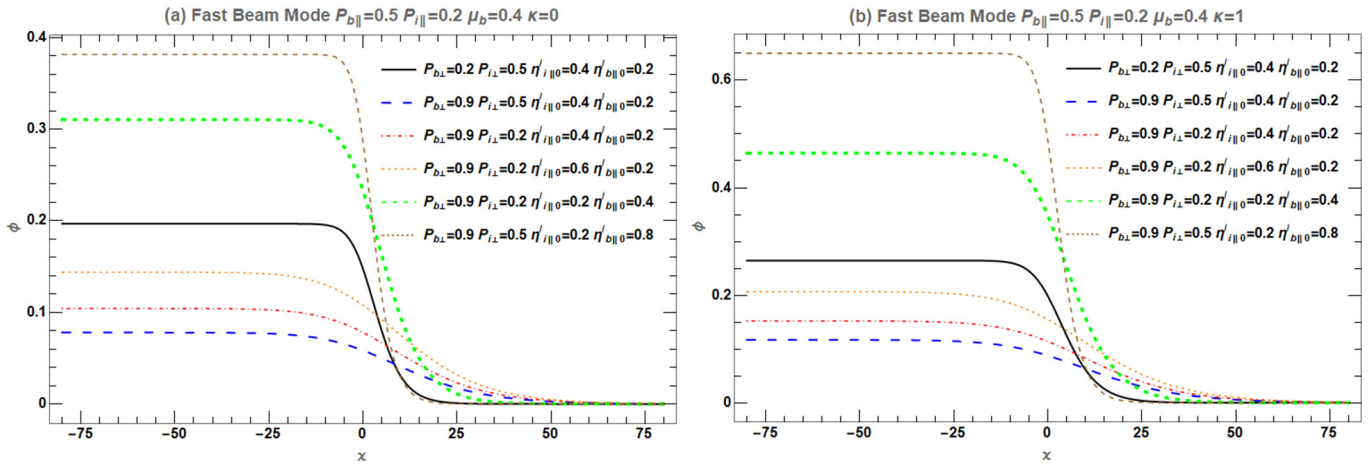


Figure 7. (a) Variation in shock wave potential profile of fast beam mode with parallel pressure of beam ions higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electrons is equal to the spin-down electrons, (b) Variation in shock wave potential profile of fast beam mode with parallel pressure of beam ions is higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electron is more than the spin-down electrons. Here, we have considered $V_b/C_s = 2$.

Figures 8 and 9 describe the variation in steady state shock potential profile of the stable beam mode for various combinations of perpendicular pressure values of positive ions and ion beams with different values of the viscosity parameter of both the ion and ion beams for the case $P_{b||} > P_{i||}$ at two different beam velocities, i.e., at $V_b = 0.6C_s$ and $V_b = 2.5C_s$. Figures 8a and 9a are plotted for the case of equal concentration of up-spin and down-spin electrons while keeping the other parameters intact. Figures 8b and 9b are plotted for the case when the concentration of up-spin electrons is higher than the down-spin electrons. As seen from Figures 8a and 9a, the shock potential reduced by increasing the beam velocity from 0.6 to 2.5, which is also depicted in the phase velocity diagram in Figures 1–3, where a clear reduction in Mach number with increasing beam velocity is seen. On the other hand, the role of electron spin on the effect of shock potential can be seen in Figures 8b and 9b, similar to those discussed in the case of Figure 7. Now, if we consider the individual curve of Figures 8a and 9), we find that the stable mode has the least potential when both the ion and ion beams are anisotropic, and the ion beams are highly viscous with the highest chosen value of viscosity parameter (lowermost brown tiny dash curve of each figure). This is because the stable mode has a decreasing nature with beam velocity as evident from the phase velocity diagram, and in this situation, if the ion beams become more viscous, the interaction of the beam ions in the direction of propagation will be more and frequent, which slows down the shock wave further. Similarly, if we consider the uppermost orange dotted curve of Figures 8a and 9a, we see that the shock potential is the highest amongst the chosen physical situation. The reason behind this phenomenon is that here, the positive ions are isotropic as well as highly viscous compared to beam ions, and thus, there is no interaction along the direction of propagation, which allows the beam-induced stable mode to propagate with higher velocity and hence with higher shock potential. This is also evident from the amount of localization achieved in

the abovementioned two cases. We see from both the figures that when the beam ions are more viscous (brown tiny dash and green thick dash curve), the transition of shock is very smooth which signifies a higher width, i.e., a better localization, whereas when the positive ions are more viscous than the beam ions, the shock transition is sharp which signifies a lower width of the shock wave, which means under this situation, the shock wave is less localized (orange dotted and red dot dashed curve).

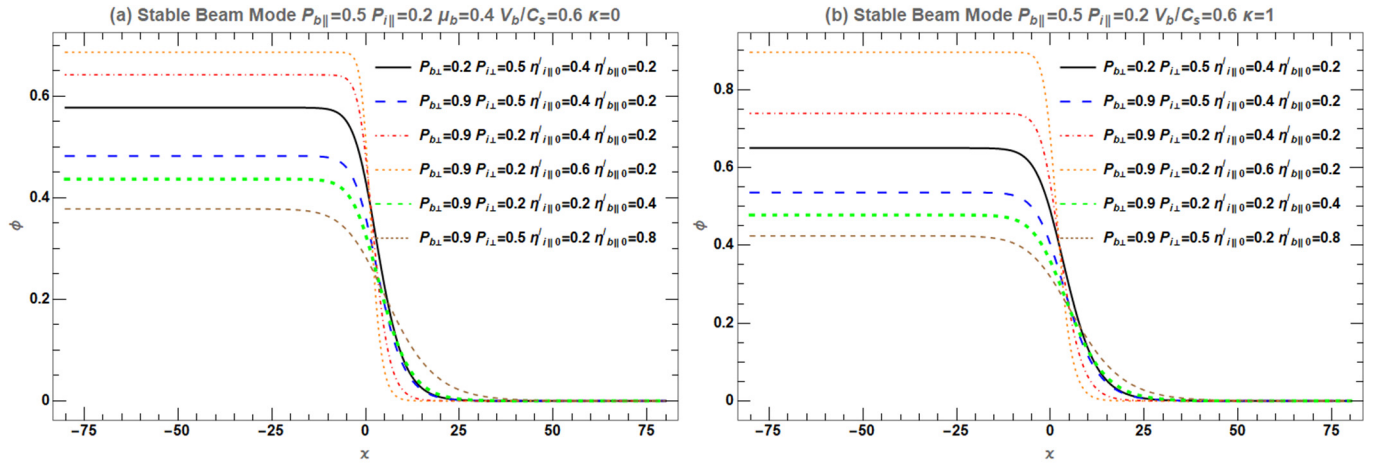


Figure 8. (a) Variation in shock wave potential profile of stable beam mode with parallel pressure of beam ions higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electron is equal to the spin-down electrons. (b) Variation in shock wave potential profile of stable beam mode with parallel pressure of beam ions is higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electron is more than the spin-down electrons.

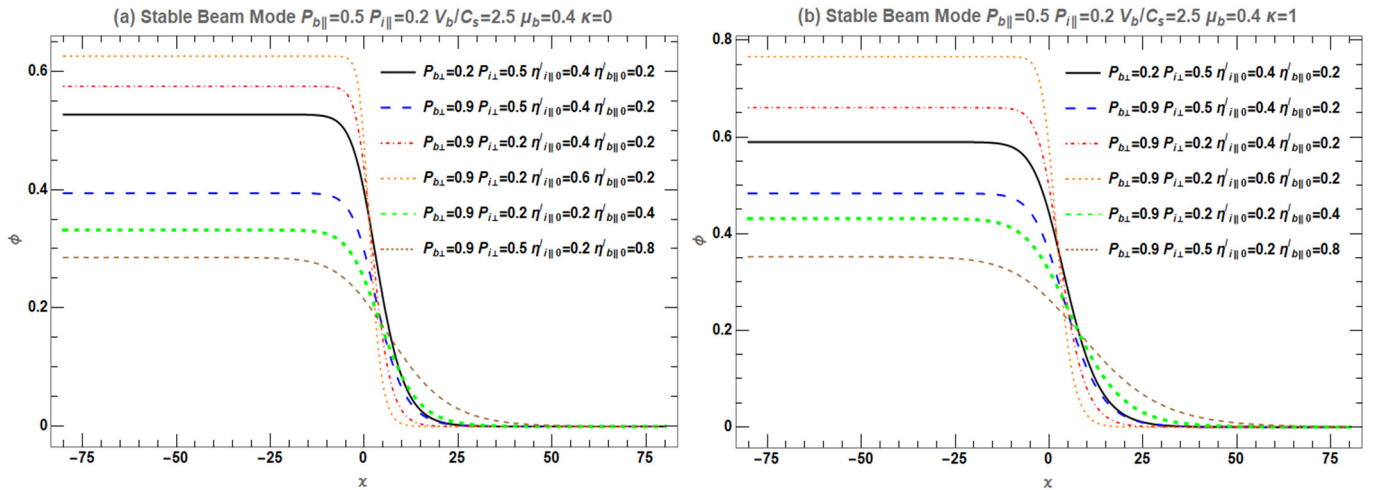


Figure 9. (a) Variation in shock wave potential profile of stable beam mode with parallel pressure of beam ions higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electron is equal to the spin-down electrons. (b) Variation in shock wave potential profile of stable beam mode with parallel pressure of beam ions higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electron is more than the spin-down electrons.

Figures 10a and 11a describe the variation of ion acoustic mode for two different beam velocities for different combinations of perpendicular pressures of ion and ion beams as

well as different viscosity parameters of ion and ion beams under the condition of equal concentration of up spin and down-spin electron. While considering the same physical parameters, the shock potential profile for the case when up spin electron concentration is more than the down-spin electron concentration is plotted in Figures 10b and 11b. If we see Figure 10a which is plotted for a lower range of beam velocity, it is clear that the shock transition is very sharp, which means in the lower range of beam velocities, the ion acoustic shock is less localized. Moreover, within this range of beam velocity, the shock potential profile is rarefactive, which is an interesting phenomenon, because we have noticed that for the range of beam velocity for which the Mach number of ion acoustic mode is decreasing (blue large dashed curve of Figures 1–3), the shock potential is rarefactive (Figure 10a), whereas for the range of beam velocities for which the Mach number of ion acoustic mode is increasing or stable, the shock potential is compressive (Figure 11a,b). Moreover, in the higher beam velocity range, i.e., in the case of Figure 11a,b, it is seen that the transition of shock is very smooth, which means at higher beam velocities, the ion acoustic shock is better localized. Another important and interesting observation is that in the lower range of beam velocity, i.e., when the Mach number of the ion acoustic mode is decreasing, the ion acoustic shock wave potential becomes compressive (Figure 10b) under the condition when the plasma system has more concentration of up-spin electron density than down-spin electron density, i.e., $\kappa = 1$ case. To understand this peculiar behaviour, we need to understand the basic mechanism of ion acoustic wave propagation. In ion acoustic wave propagation normally, electrons are not fixed; they are pulled along with ions and try to shield the electric field that arises from the bunching of ions. However, due to the electron's thermal motion, the shielding is not perfect as it can move with faster velocity being lighter than the ions and, thus, can leak into the plasma. Thus, the bunching of ions forms a region of compression which tends to expand to rarefaction due to the ion's thermal motion and the resultant electric field arising from the bunching of ions as well. The electric field is not properly shielded due to the reason mentioned above and due to inertia, the ion overshoots and thus, the region of compression and rarefaction is regenerated to form a wave. Now, if in this situation, the concentration of up-spin electrons increases than the down-spin electrons, the density gradient of electrons will help in shielding the electric field, thereby preventing the ions from going for the region of rarefaction. So, naturally, the wave's compression increases, and hence, the reason for becoming an ion acoustic wave is potentially compressive when the up-spin electron density is more than the down-spin electron density (Figure 10b). Moreover, as the phase velocity decreases in that region, this will further prevent the ion acoustic waves from going to rarefaction, thereby increasing the compression. The same nature of the variation in ion acoustic shock wave potential is noticed in the numerical simulation performed in Figure 17, described in the later section. In the same context, we can understand the increase in the compressive shock wave potential when the concentration of up-spin electron density is more than the down-spin electron density (Figure 11b) for which the wave is already compressive. Apart from this, if we see the individual curves of both the figures, we see that the shock potential is the highest when the beam ions are anisotropic but the positive ions are isotropic (orange dotted curve of each figure). This is because the ion acoustic mode is the inherent plasma mode which is not beam-induced, so in the direction of propagation when the beam ions are anisotropic but the positive ions are isotropic, there will be very less interaction which will allow the ion acoustic mode to propagate freely which in turn increases the phase velocity and hence the shock potential. On the other hand, the shock potential is the lowest when both the ion and ion beams are anisotropic (brown small dashed curve of both the figures). This is because as both of them are anisotropic, the interaction along the direction

of propagation will be more which eventually lowers the phase velocity and hence the shock potential.

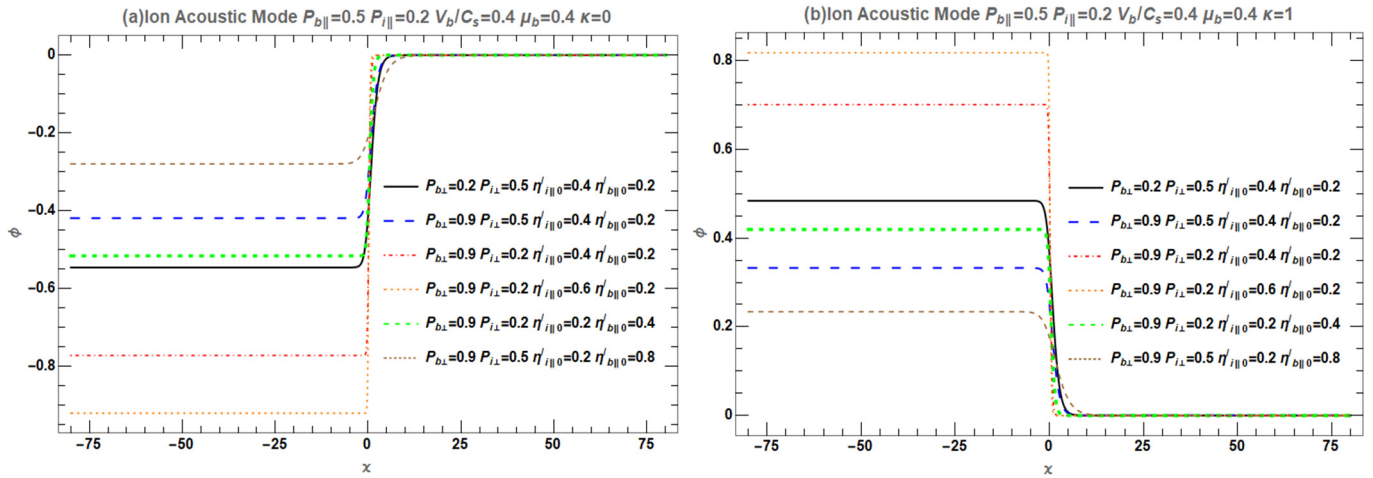


Figure 10. (a) Variation in shock wave potential profile ion acoustic mode with parallel pressure of beam ions higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electrons is equal to the spin-down electrons, (b) Variation in shock wave potential profile of Ion Acoustic mode with parallel pressure of beam ions is higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electrons is more than the spin-down electrons.

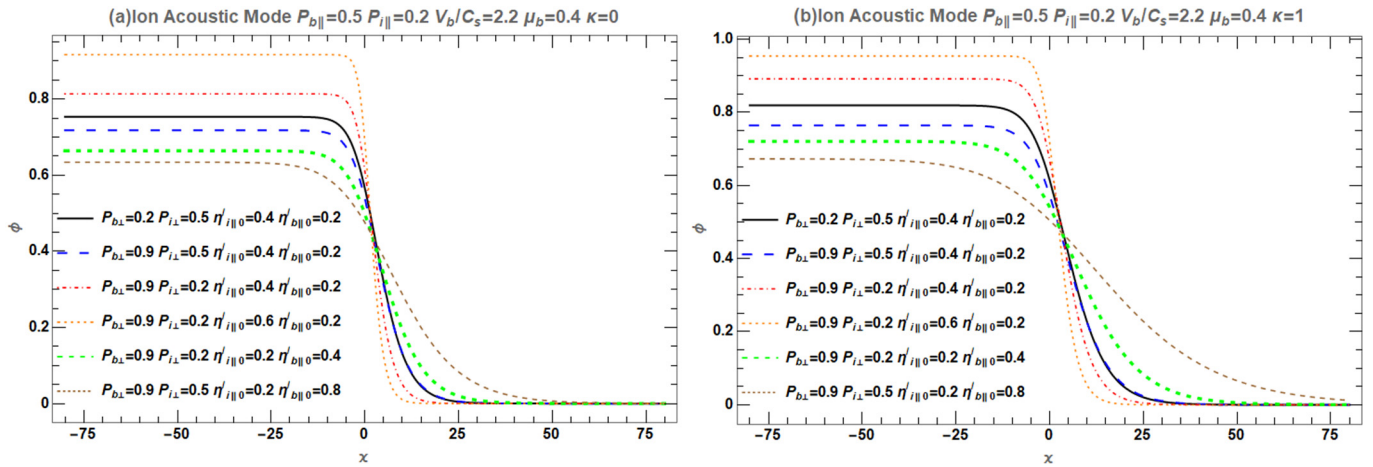


Figure 11. (a) Variation in shock wave potential profile ion acoustic mode with parallel pressure of beam ions is higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electrons is equal to the spin-down electrons for higher beam velocity, (b) Variation in shock wave potential profile of ion acoustic mode with parallel pressure of beam ions higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electrons is more than the spin-down electrons for higher beam velocity.

Figure 12 demonstrates the variation in the steady state shock profile of slow beam mode with different values of perpendicular pressure of ion and ion beams. Figure 12a,b represent the variation of slow beam mode for the two cases: Figure 12a is plotted for the case when the up-spin electron and down-spin electron density is the same, whereas Figure 12b is plotted for the case when the up-spin electron density is more than the down-spin electron density. As seen from the figure for the case $\kappa = 0$, the shock potential for the slow beam mode is rarefactive, whereas for the same case, the fast mode is compressive

(Figure 7a). This is an intrinsic property of the ion beam plasma in which it has been showed theoretically as well as experimentally that if the fast beam mode is compressive (rarefactive), the slow beam mode is rarefactive (compressive), i.e., the fast and slow beam mode are always exactly opposite to each other. If we closely observe Figure 12a, we see that the shock potential is the lowest (brown small dashed curve) when both the ion and ion beams are anisotropic and the ion beams are highly viscous. This is because as both ion and ion beams are anisotropic, the interaction of ion and ion beams along the direction of propagation will be the highest which will eventually slower the shock wave potential. On the other hand, the shock potential is the highest when the beam ions are anisotropic while the positive ions are isotropic and viscous. This is because, as it is a beam-induced mode, the beam ions being anisotropic, there will be very less interaction with the positive ions which are isotropic in the direction of propagation, and hence, the wave becomes free to move and the shock potential speeds up. The same trend of variation in shock potential can be understood for the obvious reason discussed above in the case of Figure 12a. But interestingly, the shock potential becomes compressive when the up-spin electron density is more than the down-spin electron density. The reason for this switching of shock potential can be understood from the similar context of the explanation provided in the case of Figure 10b. Moreover, though the shock potential of the slow beam mode becomes compressive under the condition of higher up-spin electron density than the down-spin electron which is similar to the propagation of fast beam mode depicted in Figure 7b, the direction of propagation becomes opposite to that described in the case of Figure 7b, and thus, the slow and the fast beam mode maintains a clear distinction between its propagation characteristics. This peculiar behaviour is also realized in the case of numerical simulation of the slow beam mode described in Figure 16. This is an intrinsic property of the ion beam plasma system where the beam-induced modes, i.e., the fast and slow beam modes always maintain different propagation characteristics.

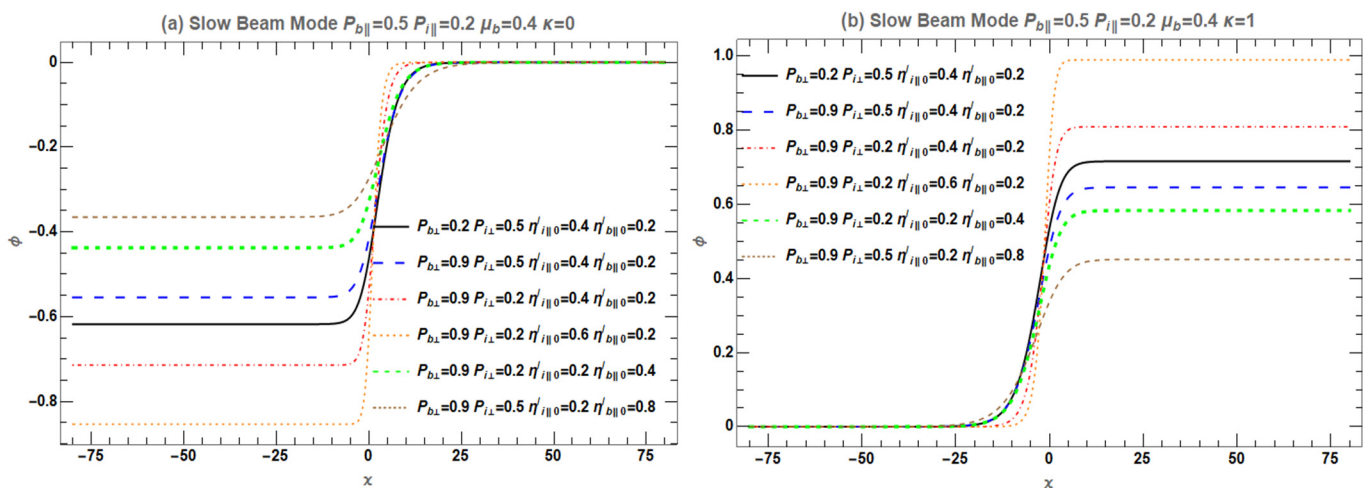


Figure 12. (a) Variation in shock wave potential profile of slow beam mode with parallel pressure of beam ions higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electrons is equal to the spin-down electrons. (b) Variation in shock wave potential profile of slow beam mode with parallel pressure of beam ions is higher than the ions for a different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electrons is more than the spin-down electrons. Here, we have considered $V_b/C_s = 2.5$.

Figure 13 demonstrates the variation in fast beam mode, slow beam mode, stable beam mode and ion acoustic mode for the higher concentration of positive ion beam, i.e., $\mu_b = 0.8$. It is to be noted that all the previous figures are plotted for the case $\mu_b = 0.4$.

So, here we see that when the ion beam density increases except in the fast beam mode, the shock potential of all other modes decreases. This is because when more and more energetic ion beams are introduced, these beams enhance the phase velocity of the fast beam mode which is induced due to strong coupling between beam ions and background ions. On the contrary, the slow beam mode becomes slower as more and more beam ions are introduced, simply because slow beam mode is generated by those ion beams which cannot couple to the background ions [63] and as the density of the ion beams increases, due to prolonged and frequent interaction, the slow beam mode becomes slower; hence, it propagates with a reduced shock potential. Similarly, we can see a diminishing shock potential with increasing beam density for the stable beam mode simply because we have seen from the phase velocity diagram that the stable beam mode has a reducing nature with beam velocity; so, as more energetic ion beams are introduced, the wave will suffer more interaction with the larger amount of ion beams which eventually makes it slow. On the other hand, the ion acoustic mode which is an inherent background ion mode suffers a large amount of interaction with increasing ion beam density and hence becomes slower. We examined the abovementioned situation for the case when the up spin electron has more density than the down-spin electron and found a similar role of electron spin on the shock wave as discussed in the previous cases.

Figure 14a describes the numerical evolution of the fast beam mode when beam ions are more viscous than the positive ions, whereas Figure 14b describes the numerical evolution of the fast beam mode when positive ions are more viscous than the beam ions in a particular time interval $\tau = 5$. For the two figures, we have chosen three sets of perpendicular pressures, in one set, we have considered $P_{b(i)\perp} \neq P_{b(i)\parallel}$ which means both positive ion and ion beam are anisotropic. In one set, we have considered $P_{b\perp} = P_{b\parallel}$ but $P_{i\perp} \neq P_{i\parallel}$, which means ion beams are isotropic but positive ions are anisotropic. In another set, we have considered $P_{b\perp} \neq P_{b\parallel}$ but $P_{i\perp} = P_{i\parallel}$, which means positive ions are isotropic but ion beams are anisotropic. If we consider the individual plots of Figure 14a, we see that if the beam ions are more viscous than the positive ions, the temporal shock pattern is oscillatory having the highest compressive shock potential for the case when both positive ions and ion beams are anisotropic (orange dotted curve) and the shock oscillates with the lowest compressive potential when the beam ions are isotropic but positive ion are anisotropic (black dot dashed curve). This is also seen in Figure 7a in the case of steady state propagation of shock wave that if the positive ions and beam ions are anisotropic with the highest value of the viscosity parameter of beam ions, and the shock propagated with the highest possible potential though the perpendicular pressure values of beam ions is different for both the figures. Thus, whether it is oscillatory or monotonic shock, the shock potential is always high when both the beam ions and positive ions are anisotropic and beam ions are highly viscous, and the reasons are of course discussed in the context of Figure 7. On the other hand, since it is a beam-induced mode, as the beam ions are isotropic, the contribution of highly viscous beam ions in the direction of propagation will be less, which eventually lowers the shock potential (black dot dashed curve). On the other hand, in Figure 14b, we have seen that the temporal evolution of the shock wave becomes monotonic for the same pressure combination when the positive ions are more viscous than the beam ions. This is quite an interesting observation from which we can conclude that a temporal evolution of oscillatory shock becomes monotonic within the same time interval if the viscosity of positive ions is more than the beam ions. Moreover, the shock potential is much lower when positive ions are more viscous than beam ions. If we consider the individual curve of Figure 14b, we see that when both the ion and ion beams are anisotropic (brown small dashed curve), the shock potential is the highest, but here, shock potential is the lowest when the beam ions are anisotropic and positive ions are isotropic. It is because

since it is a beam-induced mode, the interaction of anisotropic beam ions along the direction of propagation with highly viscous ions will be more, which makes the shock potential lower. This is another basic difference between both the figures, which clearly shows that when the beam ions are more viscous than positive ions, the isotropic (anisotropic) beam (positive ion) makes the shock potential lowest (black dot dashed curve of Figure 14a), and when the positive ions are more viscous than the beam ions, the anisotropic (isotropic) beam ions (positive ions) make the shock potential the lowest (green dot dashed curve of Figure 14b). Moreover, the shock pattern is better localized (Figure 14b) when the positive ions are more viscous than the beam ions compared to the case when beam ions are more viscous than the positive ions (Figure 14a).

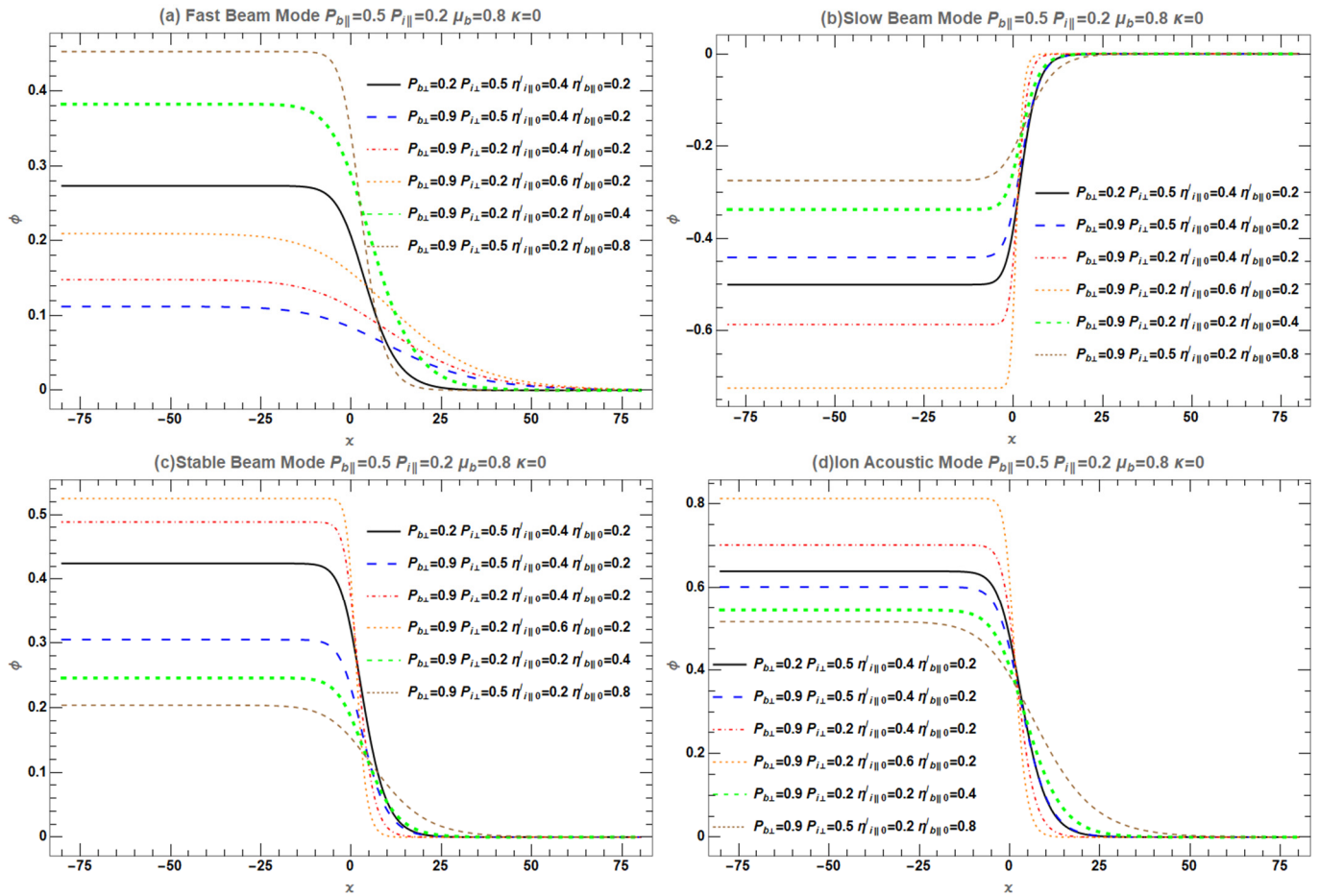


Figure 13. (a) Variation in shock wave potential profile of fast beam mode with parallel pressure of beam ions greater than the ions for different combination of perpendicular pressure anisotropy and viscosity parameter of ion and ion beams when the concentration of spin-up electrons is equal to the spin-down electrons for the case $\mu_b = 0.8$. (b) Variation in shock wave potential profile of slow beam mode with parallel pressure of beam ions greater than the ions for different combination of perpendicular pressure and viscosity parameter of ion and ion beams when the concentration of spin-up electrons is equal to the spin-down electrons for the case $\mu_b = 0.8$. (c) Variation in shock wave potential profile of stable beam mode with parallel pressure of beam ions greater than the ions for different combination of perpendicular pressure and viscosity parameter of ion and ion beams when the concentration of spin-up electrons is equal to the spin-down electrons for the case $\mu_b = 0.8$. (d) Variation in shock wave potential profile of Ion Acoustic mode with parallel pressure of beam ions greater than the ions for different combination of perpendicular pressure and viscosity parameter of ion and ion beams when the concentration of spin-up electron is equal to the spin-down electrons for the case $\mu_b = 0.8$. Here, we have considered $V_b/C_s = 2.5$.

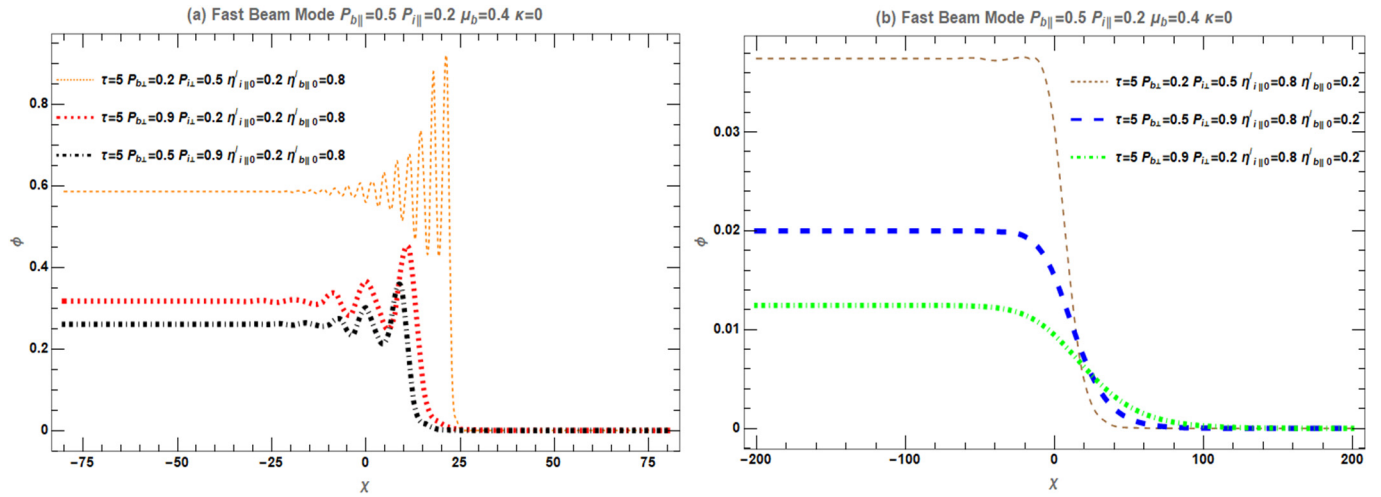


Figure 14. (a) Numerical evolution of shock wave potential profile of fast beam mode with parallel pressure of beam ions higher than the ions for different combinations of perpendicular pressure ion and ion beams with viscosity of beam ions higher than the positive ions, (b) Variation in shock wave potential profile of fast beam mode with parallel pressure of beam ions higher than the ions for different combination of perpendicular pressures of ion and ion beams with viscosity of positive ions higher than the beam ions. Here, we have considered $V_b/C_s = 2.5$.

Figure 15 describes the temporal evolution of the slow beam mode for the same combination of perpendicular pressures as well as the viscosity parameter of ions and ion beams as considered in Figure 14. Here, as already described in the previous cases, there is always a distinction between the fast and slow mode, i.e., if the fast mode is compressive (rarefactive), then the slow beam mode is rarefactive (compressive); this is reflected in the numerical simulation also, which appears as rarefactive as for the same conditions, the fast mode is compressive (Figure 14a). Here, as seen in Figure 15a, the shock potential suffers the highest rarefactive potential when both the beam ions and positive ions are anisotropic. Here, the interesting feature is that though the ion beams are anisotropic, the perpendicular pressure of beam ions is less than the parallel ion beam pressure, so the interaction of highly viscous beam ions in the direction of propagation will be very less, which will eventually allow the slow beam mode to travel with maximum rarefactive potential. The same can be noticed from the other two graphs where we see that as the perpendicular pressures of beam ions increase (black dot dashed to red dashed curve), the interaction of viscous beam ions in the direction of propagation of the shock wave increases, which further reduces the rarefactive shock potential. Here again, we see from Figure 15b that when the viscosity of positive ions is more than the beam ions, the temporal oscillatory shock potential turns out to be a monotonic one. Here again, we see that the shock pattern is better localized (Figure 15b) when the positive ions are more viscous than the beam ions compared to the case when beam ions are more viscous than the positive ions (Figure 15a).

Figure 16 demonstrates the numerical evolution of the shock pattern of slow beam mode for the case when up spin electron density is more than the down-spin electron for the same range of perpendicular pressure combinations as considered for Figures 14 and 15. Here, we have considered $V_b = 2.5C_s$, i.e., a situation $V_b > C_s$. As seen from Figure 16, the propagation direction as well as the magnitude of the shock potential changes dramatically under the influence of a higher concentration of up spin electron density. In our earlier observation, we have reported in Refs. [1,4,65] that the slow mode is rarefactive (compressive) when the fast mode is compressive (rarefactive). Surprisingly, here, the slow beam mode maintains a clear difference from the fast beam mode. But interestingly, though the slow mode (Figure 16) and the fast mode (Figure 7) are compressive, the directions

of propagation of both the modes are exactly opposite to each other. This is an interesting observation which occurs only due to the consideration of the spin contribution of degenerate electrons. This is another intrinsic property of the ion beam plasma system that when the up-spin electron density is more than the down-spin electron density, the direction of propagation of the slow beam mode and fast beam mode changes. This is the first observation of such a phenomenon in ion beam plasma. It is to be noted the change in the direction of propagation of the slow beam mode and fast beam mode happens only in the case when the up-spin electron density is more than the down-spin electron density, i.e., $\kappa = 1$, whereas in the case of $\kappa = 0$, i.e., when the up-spin electron density and down-spin electron density is equal, the slow beam mode appears as rarefactive for which the fast beam mode is compressive, which is confirmed both analytically and numerically (Figures 7a, 12a, 14a and 15a). As of now, we have understood that the probable reason behind this phenomenon is that when $V_b > C_s$, the beam and background ions cannot couple strongly. As a result, the beam ions can support ion acoustic waves whose phase velocities are shifted by the beam velocities [62,63]; thus, the fast beam mode becomes faster and the slow beam mode becomes slower. In this situation, if the up-spin electron density becomes higher than the down-spin electron, the natural density gradient of spin-polarized electrons will try to reduce the amount of rarefaction for the obvious reason discussed in the case of Figure 10b, and as a result, the wave potential might switch from a rarefactive to compressive one. Apart from this, in ion beam plasma, there is an intrinsic difference between the fast beam mode and the slow beam mode which is maintained by the plasma system itself, irrespective of any physical situation considered. So, since the shock potential of both the beam modes becomes compressive, the direction of propagation of both modes becomes opposite to each other. Thus, the ion beam plasma system naturally maintains a basic difference between fast beam mode and slow beam mode either in terms of the polarity of their potential or in terms of the direction of propagation. The former one is well established theoretically, experimentally and numerically, but the latter one is realized here in this investigation only with numerical simulation, which is yet to be explored experimentally. This is also confirmed in the case of steady state shock propagation described in Figure 12b where it is seen that when the up-spin electron density is more than the down-spin electron density, the shock potential becomes compressive but the direction of the shock wave propagation changes to maintain a difference between fast beam mode (Figure 7b) and slow beam mode shock wave propagation. This is an absolutely new and unique intrinsic feature of spin-polarized ion beam quantum magnetoplasma which we think might be explored more in detail if we consider the proper kinetic treatment of the current plasma model, taking into account the wave particle interaction, which is necessary to understand such peculiar behaviour. However, at present, it is beyond the scope of this study, although we have already started exploring the reason behind this phenomenon.

Figure 17 describes the variation in numerical evolution of ion acoustic mode for the two different cases, i.e., $\kappa = 0$ and $\kappa = 1$. Here again, we have seen that when the up-spin electron density is more than the down-spin electron density (Figure 17b), the polarity of the shock potential changes from rarefactive to compressive. If we closely observe the phase velocity diagram in the beam velocity range which we have considered here, i.e., $V_b \approx 0.4C_s$, we see that the Mach number is decreasing, which means the phase velocity is decreasing, and if in this condition, the concentration of up-spin electron increases more than the down-spin electron, the density gradient of electrons will further reduce the amount of rarefaction (which is well explained in the context of Figure 10b); hence, there is a possibility of shock potential changing to compressive. This peculiar behaviour of ion acoustic mode was not observed in the ion beam plasma system. On the other hand, if we analyze the individual curve of each figure, we see that the ion acoustic shock has

the highest rarefactive (compressive) potential, when the beam ions are anisotropic and viscous and positive ions are isotropic and less viscous than the beam ions. This is because the ion acoustic mode is an inherent plasma mode and not beam-induced, so when the beam ions are anisotropic in the direction of propagation and positive ions are isotropic, there will be very few interactions between positive ions and beam ions, which eventually allows the ion acoustic mode to propagate with the highest potential. In the same context, the ion acoustic shock propagates with the lowest shock potential when the positive ions are anisotropic in the direction of propagation, i.e., in the perpendicular direction, which is the case for the black dot dashed curve of each figure.

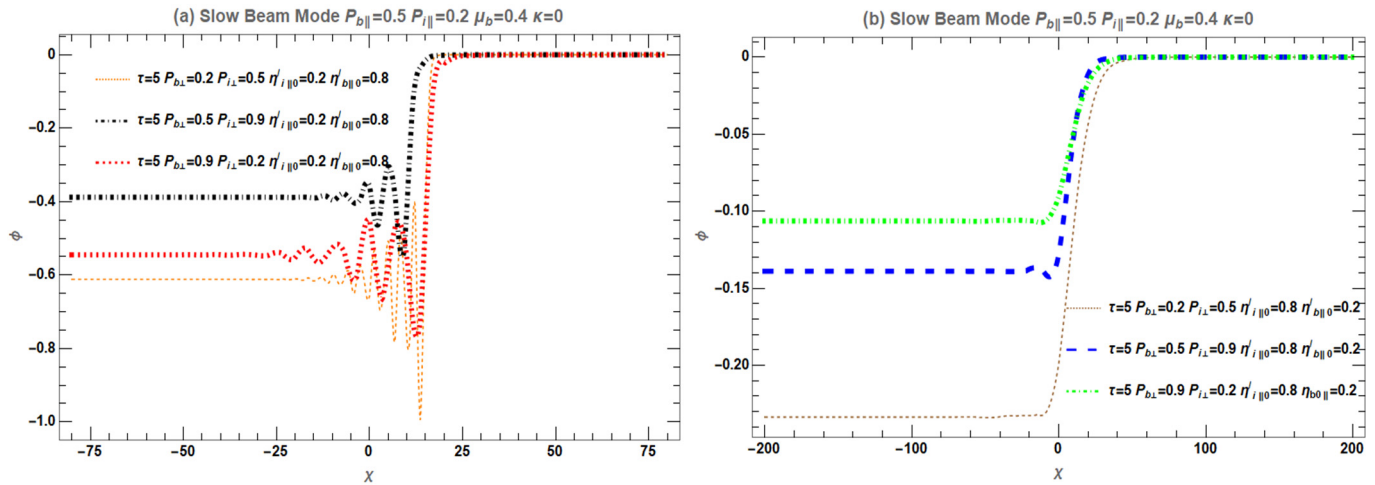


Figure 15. (a) Numerical evolution of shock wave potential profile of slow beam mode with parallel pressure of beam ions higher than the ions for different combinations of perpendicular pressure ion and ion beams with the viscosity of beam ions higher than the positive ions, (b) Variation in shock wave potential profile of slow beam mode with parallel pressure of beam ions higher than the ions for different combination of perpendicular pressures of ion and ion beams with viscosity of positive ions higher than the beam ions. Here, we have considered $V_b/C_s = 2.5$.

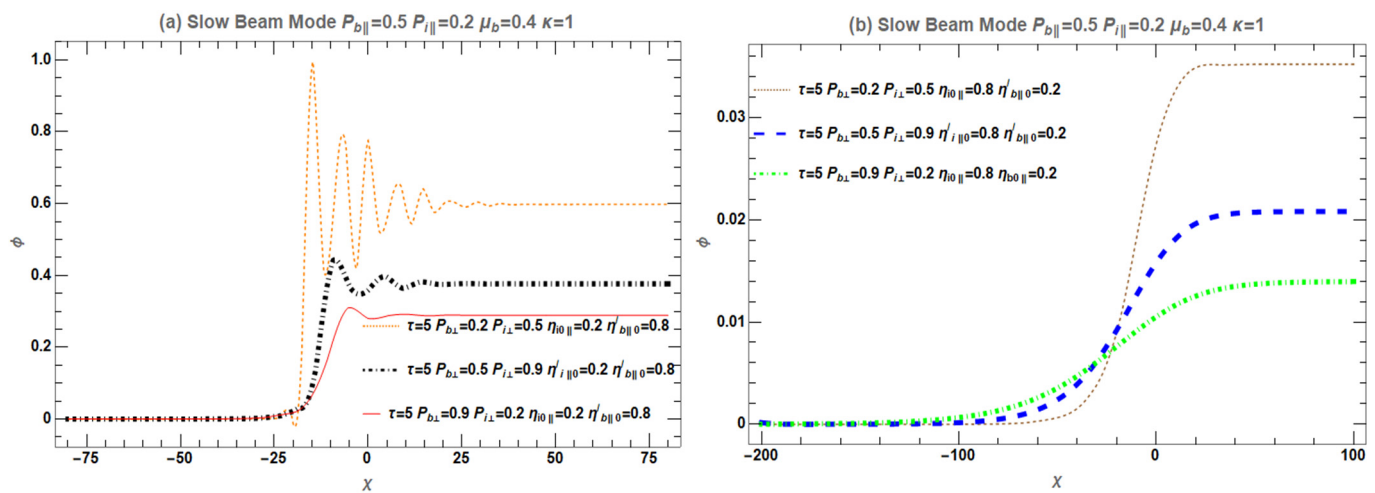


Figure 16. (a) Numerical evolution of shock wave potential profile of slow beam mode with parallel pressure of beam ions higher than the ions for different combinations of perpendicular pressure and viscosity parameter of ion and ion beams with viscosity of beam ions higher than the positive ions. (b) Variation in shock wave potential profile of slow beam mode with parallel pressure of beam ions higher than the ions for different combination of perpendicular pressure of ion and ion beams with viscosity of positive ions higher than the beam ions. Here, we have considered $V_b/C_s = 2.5$.

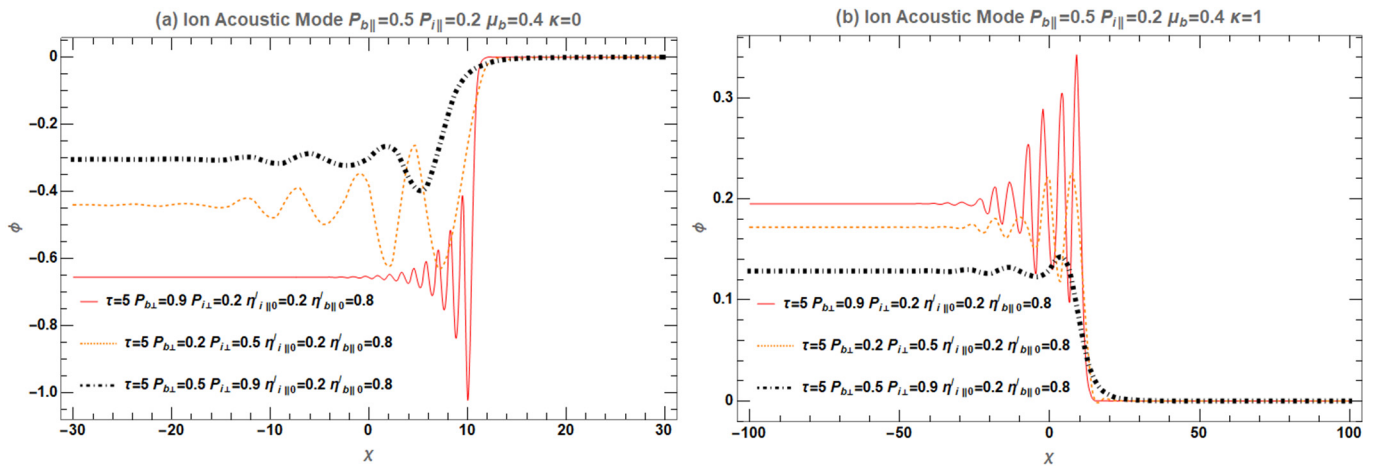


Figure 17. (a) Numerical evolution of shock wave potential profile of ion acoustic mode with parallel pressure of beam ions higher than the ions for a different combination of perpendicular pressure and viscosity parameter of ion and ion beams with the viscosity of beam ions higher than the positive ions. (b) Variation in shock wave potential profile of ion acoustic mode with parallel pressure of beam ions higher than the ions for a different combination of perpendicular pressure of ion and ion beams with the viscosity of positive ions higher than the beam ions. Here, we have considered $V_b/C_s = 0.4$.

Figure 18 describes the evolution of stable beam mode for two different cases. In Figure 18a, we have considered the $\kappa = 0$ case for which the stable beam mode propagates with subsonic speed (Figures 1a, 2a and 3a), and in Figure 18b, we have considered the $\kappa = 1$ case for which the value of parallel pressures of ion and ion beams are higher than all the other cases considered till now, and the ion beam density is also high (Figure 3b) for which the stable beam mode propagates with supersonic speed. Though Figure 3b is considered for the case $\kappa = 0$, we have checked for the case $\kappa = 1$, for which we found that the stable beam mode also propagates with supersonic speed. The perpendicular pressure range of ion and ion beams is same as Figures 14–16. This is another interesting observation considering the fact that, in our previous cases, we found that when the concentration of up-spin electrons is more than the down-spin electrons, the shock potential increases, but here, we see that the shock potential decreases for the same case when the stable mode propagates with supersonic speed. This is primarily because when the stable beam mode propagates with a supersonic speed that also has higher ion beam density, the probability of higher and more frequent charged particle collision may result in damping of the phase velocity and hence the possibility of the wave propagating with less phase velocity or otherwise lower shock potential. Moreover, the increase in the concentration of up-spin and down-spin electrons will also make the wave speed up which, in turn, increases the possibility of frequent charged particle collision, which further helps in reducing the shock potential. Now, if we consider the individual plot of Figure 18a, we see that the shock potential of the stable beam mode when it propagates in the subsonic range is the highest when beam ions are anisotropic (red dot dashed curve) for which the positive ions are isotropic as well as more viscous than the beam ions. This is because since beam ions are highly anisotropic in the perpendicular direction with isotropic viscous positive ions, the interaction of ion beams in the direction of propagation will be less, which makes the wave free to propagate with higher velocity. On the other hand, the shock potential is the lowest when the beam ions are isotropic but positive ions are anisotropic (blue thin curve) because since beam ions are isotropic and the stable beam mode is a beam-induced mode, the involvement of less ion beam in the direction of propagation will eventually make the shock potential the lowest. On the other hand, when both the beam ion and positive ions are anisotropic (brown small dashed curve), the shock potential is in the intermediate range

because the interaction among them will make the shock wave propagate with a velocity lower than when the beam ions are highly anisotropic (red dot dashed curve) and higher than when the beam ions are isotropic (blue thin curve). On the other hand, if we analyze the individual curve of Figure 18b, we see that the shock propagates with the highest (red dot dashed curve) potential when the beam ions are isotropic and lowest (blue thin curve) when the beam ions are anisotropic. This is interesting and is opposite to the case discussed in Figure 18a which is very much obvious. As more and more beam ions are involved (when beam ions are anisotropic) in the direction of propagation, i.e., in the perpendicular direction, the interaction will be more, which will eventually lower the phase velocity and hence the shock potential (blue thin curve). However, when the beam ions are isotropic, i.e., when less amount of beam ions are involved in the direction of propagation, the interaction will be less, which will eventually make the phase velocity higher and hence the shock potential higher (red dot dashed curve).

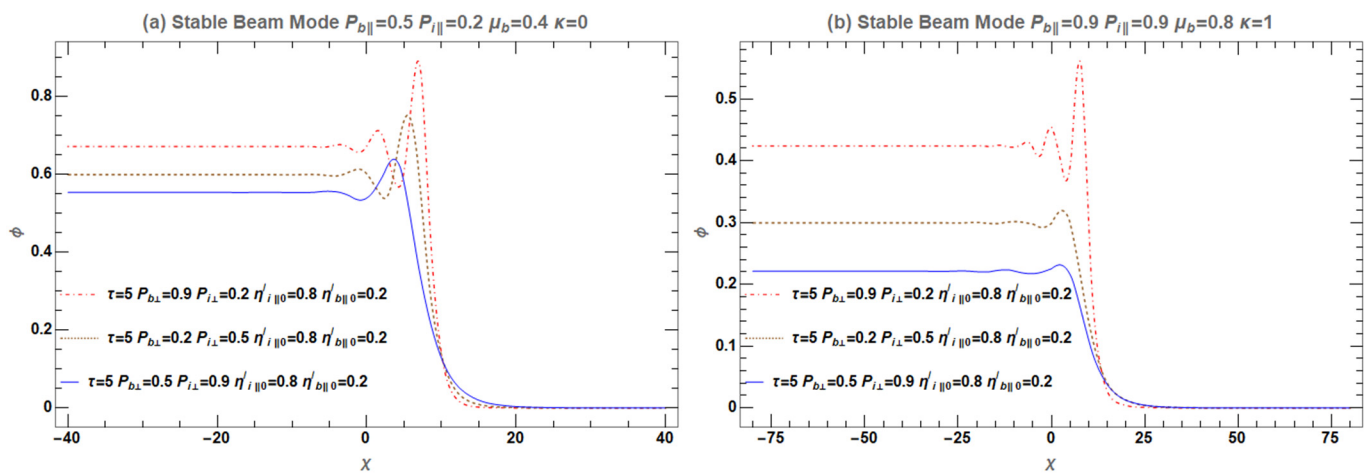


Figure 18. (a) Numerical evolution of shock wave potential profile of stable beam mode with parallel pressure of beam ions higher than the ions for different combinations of perpendicular pressure of ion and ion beams with viscosity of positive ions higher than the beam ions. (b) Numerical evolution of shock wave potential profile of stable beam mode with higher parallel pressure of beam ions and the positive ions for different combination of perpendicular pressure of ion and ion beams with viscosity of positive ions higher than the beam ions. Here, we have considered $V_b/C_s = 2.5$.

Figure 19 describes the numerical evolution of the shock potential profile of stable beam mode in a similar context to Figure 18, except here, we have considered that the viscosity of beam ions is higher than the positive ions. The basic difference between the two figures is that the shock potential becomes monotonic when beam ions have higher viscosity than the positive ions, whereas it is oscillatory when the positive ions are more viscous than the beam ions. On the other hand, the shock potential is much lower when the beam ions are more viscous than the positive ions. Apart from this, another interesting observation is that monotonic shock propagation is better localized than oscillatory shock propagation. Now, if we try to understand the physics behind this phenomenon, we see that the phase velocity of the stable mode decreases with increasing beam velocity (Inset of Figures 1–3), and if in this situation, the viscosity of the beam ions increases, the interaction of slow and highly viscous beam ions will be more in the direction of propagation of the wave, which will eventually lower the phase velocity and hence the shock potential than when the case beam ions are less viscous (Figure 18a). This phenomenon is more prominent in the case of Figure 19b. From the physical situation of Figure 19b, we can understand that though the stable mode is supersonic in that range, its phase velocity is still decreasing with beam velocity (Inset of Figure 3b), but it is higher than when the stable mode is subsonic;

thus, the possibility of frequent charged particle interaction increases in the direction of propagation of the stable beam mode which eventually lowers the shock potential even further, and that is exactly the situation described in the figure. On the other hand, the prolonged interactions of highly viscous beams make the shock wave better localized as well as more stable, and that is why the shock propagation becomes monotonic compared to the case of oscillatory shock propagation when the beam ions are less viscous than the positive ions (Figure 18a,b). The variation in the individual plots of Figure 19a,b is similar to Figure 18a,b and thus can be understood in the similar context of the explanation provided in the case of Figure 18a,b. Plasma physics is uncovering the inner workings of the Sun and stars, planetary ionospheres and magnetospheres, interstellar space, and fascinating astrophysical objects such as black holes and their jets, pulsars, magnetars, and neutron stars. The most important practical applications of plasmas lie in the future, largely in the field of power production. The major method of generating electric power has been to use heat sources to convert water to steam, which drives turbogenerators. A total of 99.9% of the observable universe—stars, nebulae, comet tails—is glowing plasma. A bolt of lightning is plasma, as are neon signs and fluorescent lights. Plasma is both ordinary and extraordinary, which is why so many scientists, including those at the PSFC, are studying it. Plasma is also where fusion happens. The width and amplitudes of the solitary waves and shocklets are significantly influenced by the presence of quantizing magnetic fields, trapped/untrapped electrons and ion-thermal corrections. The present results may prove useful to understand the self-steepening phenomenon and wave breaking of solitary waves in quantized dense plasmas, where strong magnetic fields are present.

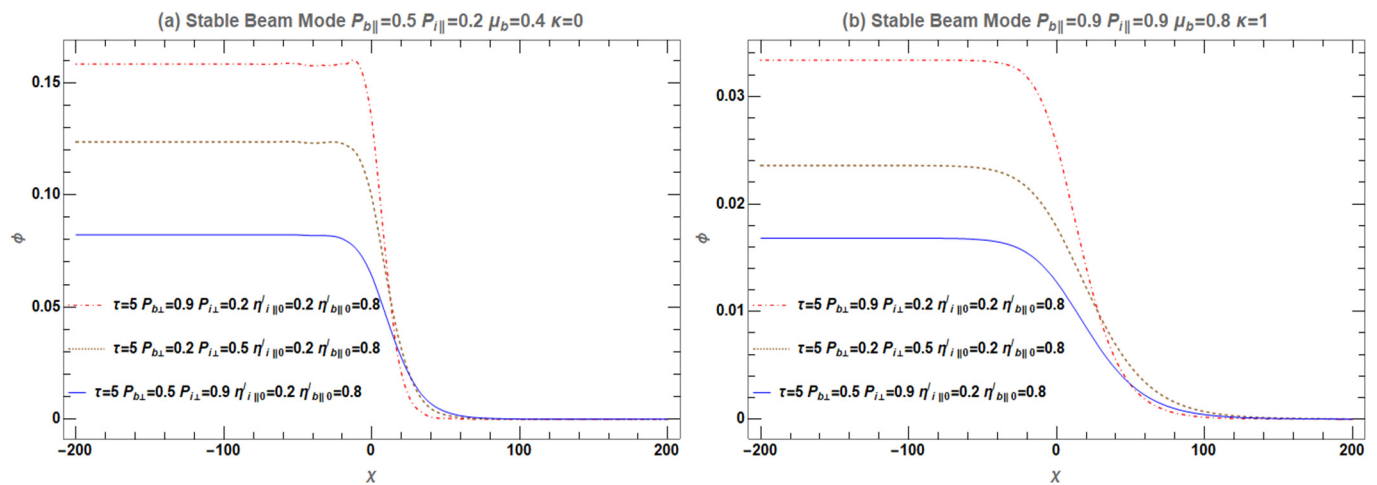


Figure 19. (a) Numerical evolution of shock wave potential profile of stable beam mode with parallel pressure of beam ions higher than the ions for different combinations of perpendicular pressure of ion and ion beams with viscosity of beam ions higher than the positive ions. (b) Numerical evolution of shock wave potential profile of stable beam mode with equal and higher parallel pressure of both the positive ions and beam ions for different combination of perpendicular pressure of ion and ion beams with viscosity of beam ions higher than the positive ions. Here, we have considered $V_b/C_s = 2.5$.

5. Conclusions

The characteristics of different modes of ion beam plasma system in an anisotropic quantum magnetoplasma under the influence of spin density polarization ratio of spin-up and spin-down electrons were studied. The properties of shock wave carried by different modes of ion beam plasma system were observed under the effect of different physical parameters relevant to the laboratory as well as space and astrophysical plasma environment. A new beam-induced mode in ion beam quantum plasma was observed with the inclusion of the pressure anisotropy effect of the ion and ion beams as well as

the spin density polarization effect of degenerate electrons. This is the first observation of this new beam-induced mode in an ion beam plasma system. This new beam-induced stable beam mode is subsonic (supersonic) for a lower (higher) range of parallel pressure of ion and ion beams irrespective of any spin density polarization ratio. It is observed that in such plasmas, the shock wave potential of beam-induced modes as well as inherent ion acoustic modes is extremely sensitive to the viscous nature of both positive ion and the beam ions as well as to the spin density polarization ratio of up-spin and down-spin electrons. The beam-induced stable mode has an oscillatory (monotonic) propagation characteristic when the beam ions are less (more) viscous than the positive ions. Apart from this, we have also observed for the first time that the inherent ion acoustic mode is affected by the ionic pressure anisotropy and spin density polarization ratio. The shock wave potential of ion acoustic and slow beam mode changes from rarefactive to compressive when the concentration of up-spin electron density is more than the down-spin electron density. Apart from the shock potential, the localization of the shock wave is also affected by the viscosity parameter of the positive ions, and it is observed that the shock wave is better localized when positive ions are more viscous than the beam ions. All the numerical and analytical results are in close agreement with each other. The present study may be helpful to understand the basic nature of wave propagation in the laboratory as well as space and astrophysical plasma environment where such condition arises, the details of which are discussed in the introduction. The experimental investigation is an interesting one to observe. In our previous experimental observation [64], we observed different modes of ion beam plasmas in a classical dusty plasma. Later [1–4], we investigated the features of ion beam plasma in magnetically quantized degenerate plasma where the ion–ion instability was not observed probably due to the inclusion of the trapping effect. The present observation opens the possibility of investigating the instabilities associated with this type of plasma environment for our companion researchers, as from the very preliminary observation, we have seen that the required beam velocity for the typical ion–ion instability varies depending on the anisotropic pressure combinations and spin density polarization ratio. Also, the detection of new beam-induced stable mode provides another opportunity to study its unique features. Apart from this, the behaviour of ion acoustic mode under different conditions of ionic pressure anisotropy and spin density polarization ratio within the lower range of beam velocity (typically less than 1) is quite interesting. But to understand the underlying mechanism, we need a specific kinetic treatment as fluid theory is not sufficient to unveil the lower range of beam velocity. This is also confirmed in earlier reports that the beam velocity range for the typical ion–ion instability is $0.7C_s \leq V_b \leq 2$, and below the lower limit, i.e., $0.7C_s$, the fluid theory is not adequate [62]. Moreover, the behaviour of slow beam mode with different values of ionic pressure anisotropy and spin density polarization ratio is also unprecedented, which can only be understood with a proper kinetic treatment as the kinetic treatment includes wave–particle interaction. But a detailed kinetic treatment at present is beyond the scope of this manuscript. We are working on the details of all the new observations reported here from the viewpoint of kinetic treatment of the plasma situation described here, and this will be communicated elsewhere. The results of the present investigation may be applied to study wave propagation in astrophysical plasma environments where the spin of electrons has to be included, such as white dwarfs, neutron stars, quark–gluon plasma, etc. Also, this study may be useful to study the energy propagation in spin-polarized plasma [66], as well as to understand the basic features of laser produced quantum plasma [65], plasmonic devices [67], etc.

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B.P.; resources, A.N.D., D.M. and J.M.; data curation, M.K.D.; writing—original draft preparation, M.K.D., A.N.D., J.M. and K.H.M.; writing—review and editing, M.K.D., A.N.D., J.M. and K.H.M.; visualization, M.K.D. and A.N.D.; supervision, M.K.D.; project administration, M.K.D.; funding acquisition, J.M. and K.H.M. All authors have read and agreed to the published version of the manuscript.

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